Grade IX

Lesson : 10 CIRCLES

Objective Type Questions

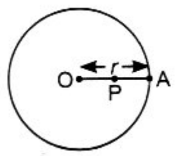
I. Multiple choice questions 10.1,2,3

1. Given a circle of radius r and with center O, A point P lies in a plane such that OP < r. Then point P lies

a) in the interior of the circle   b) on the circle

c) in the exterior of the circle   d) cannot say

d) cannot say



Sol: Let a point A lies on the circle as shown in the figure

Then OA = r and OP < r

$\Rightarrow$ OP < OA

This shows that point P lies in the interior of the circle.

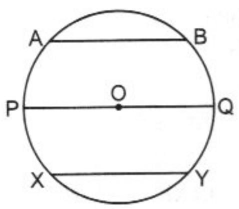
$\therefore$ Correct option is (a)

2. Given a circle with centre 0 and chords AB, PQ and XY. Points P, Q and O are collinear and radius of a circle is 6 cm. Then mark the correct option.

a) AB = XY = 3cm   b) AB = 6cm = XY   c) PQ = 6 cm   d) PQ = 12 cm

SincePoints P, Q and O are collinear and O is centre of a circle,

PQ is a diameter of a circle



$\Rightarrow$ PQ =2 x radius

$\Rightarrow$ PQ = 2 x 6 = 12 cm

$\therefore$ Correct option is (d)

3. Given a circle with centre O and smallest chord AB is of length 3 cm, longest chord CD is of length 10 cm and chord PQ is of length 7cm then radius radius of the circle is

a) 1.5 cm 2) 6cm c) 5cm d) 3.5 cm

Sol. c) 5cm

4. The region between an arc and the two radii, joining the centre to the end points of the arc is called a /an

a) sector b) segment c) semicircle d) arc

Sol. a) sector

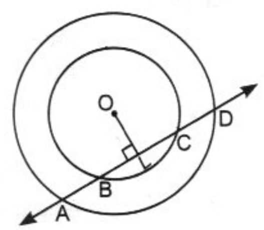
5. In how many parts a plane can divide a circle if it intersect perpendicularly?

a) 2 parts b) 3 parts c) 4parts d) 8 parts

Sol. a) 2 parts

6. Given two concentric circles with centre O, A line cuts the circles at A,B,C,D respectively. If AB = 10 cm then length CD is

a) 5cm b) 10cm c) 7.5cm d) 20 cm



Sol. Draw OL ⊥ AD

As OL ⊥BC, so BL = LC ---(i)

Similarly, AL = LD -----(ii)

Subtracting (i) from (ii) , we get

AL – BL = LD – LC

⇒ AB = CD

⇒ CD = 10 cm

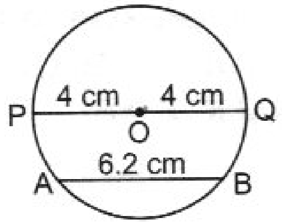
∴ Correct option is (b)

7. Through three collinear points, a circle can be drawn

a) True b)False

Sol. because a circle through two point cannot pass through a point which is collinear to these two points.

8. Justify the statement: A circle of radius 4cm can be drawn through two points A and B, such that AB=6.2cm.



Sol : It is true that a circle of radius 4cm can be passed through two point A and B, where AB = 6.2 cm

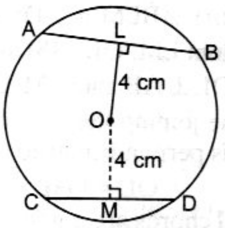
If we draw a circle of radius 4 cm, the length of longest chord, i.e. diameter = 8 cm

Such diameter > AB = 6.2 cm Hence a chord of 6.2 cm can be drawn in a circle as shown in the figure.

II. Multiple choice questions

1. Given a chord AB of length 5 cm, of a circle with centre O. OL is perpendicular to chord AB and OL = 4 cm. OM is perpendicular to chord CD such that OM = 4 cm. Then CM is equal to

a) 4cm b) 5cm c) 2.5 cm d) 3 cm



Sol: Since OL = OM = 4 cm, so

AB = CD

(∵ Chords equidistant from the centre of a circle are equal in length)

⇒ CD = 5 cm

Since the perpendicular drawn from the centre of a circle to a chord bisects the chord, so

CM = MD = $\frac{1}{2}$ CD $\Rightarrow$ CM = 2.5 cm

∴ Correction option is (c)

2. Justify your statement

"The angles subtended by a chord at any two points of a circle are equal"

The angles subtended by a chord at any two points of a circle are equal if both the points lie in the same segment (major or minor), otherwise they are not equal.

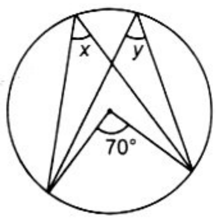
3. Justify your statement

"Two chords of a circle of lengths 10 cm and 8 cm are at the distances 8cm and 3.5 cm respectively from the centre"

The statement is not correct because the longer chord will be at smaller distance from the centre.

III. Multiple choice questions

1. In the given figure, value of y is



a) 35^0 b) 140^0 c) $70^0 + x$ d) 70^0

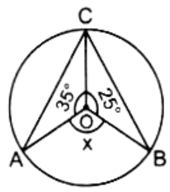
Sol : The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle. So,

Y = $\frac{1}{2}$ x $70^0 = 35^0$

∴ Correct option is (a)

2. In figure, O is the centre of the circle. The value of x is

a) 140^0 b) 60^0 c) 120^0 d) 300^0



Sol. We have ∠AOC + ∠BOC + ∠AOB = 360^0

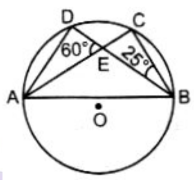
(∴ Angle at the centre of a circle)

⇒ $35^0 + 25^0 + x = 360^0 \Rightarrow x = 300^0$

∴ Correct option is (d)

3. In the given figure, O is the centre of the circle, ∠CBE = 25^0 and ∠DEA = 60^0. The measure of ∠ADB is

a) 90^0 b) 85^0 c) 95^0 d) 120^0



Sol. We have ∠DEA = ∠CEB = 60^0 (Vertically opposite angles)

Using angle sum property of triangle in Δ CEB we have

∠CEB + ∠CBE + ∠ECB = 180^0

⇒ $60^0 + 25^0$ + ∠ECB = 180^0

⇒ ∠ECB = 95^0

Now, ∠ADB = ∠ACB

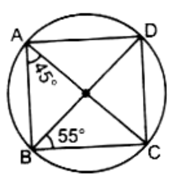
(∵ Angles in the same segment of a circle are equal)

⇒ ∠ADB = 95^0

∴ Correct option is (C)

4. In the given figure, ∠DBC = 55^0, ∠BAC = 45^0 Then ∠BCD is

a) 45^0) 55^0, c)100^0 d) 80^0



Sol : We have $\angle$ BAC = $\angle$ BDC

($\because$ Angles in the same segment of a circle are equal)

$\Rightarrow \angle$ BDC = 45^0

Using angle sum property of triangle in Δ BDC, we get

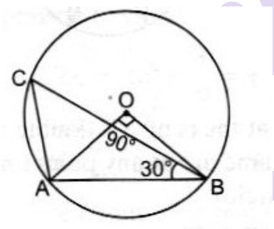
$\angle$ DBC + $\angle$BDC + $\angle$BCD = 180^0

$\Rightarrow \angle$ BCD = 80^0

$\therefore$ Correct option is (d)

5. In figure $\angle$ AOB = 90^0 and $\angle$ABC = 30^0 then $\angle$CAO is equal to

a) 30^0 b) 45^0 c) 90^0 d)60^0



We have $\angle ACB = \frac{1}{2}\angle AOB$

$= \frac{1}{2} \times 90^0 = 45^0$

Using angle sum property of triangle in ΔCAB, we get

ΔCAB = 105^0

Since OA = OB ($\because$ Radii of the circle)

$\Rightarrow \angle$ OBA = $\angle$ OAB

Using angle sum property of ΔAOB, we get

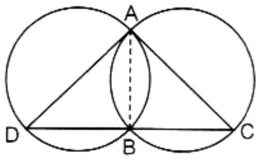
$\angle$ OAB = 45^0

Now, $\angle$ CAO = $\angle$CAB - $\angle$ OAB

= 105^0 - 45^0 = 60^0

$\therefore$ Correct option is (d)

6. Two circles intersect at two points A and B, AD and AC are diameters of the two circles. Prove that B lies on the line segment DC.



Given : Two circles intersect at A and B. AD and AC are diameters

To prove : B lies on DC

Construction : Join AB

Proof : AD is the diameter of a circle

∴ ∠ ABD = 90^0 --(i) (Angle in a semicircle)

AC is the diameter of another circle

∴ ∠ ABC = 90^0 --(ii) (Angle in a semicircle)

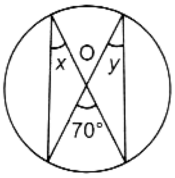
Adding (i) and (ii)

∠ ABD + ∠ ABC = 90^0 + 90^0 = 180^0

∴ These two angles from a liner pair angles

⇒DBC is a line, Hence poin B lies on line segment DC

7. In the given figure, find the value of x *and* y where O is the centre of the circle



Y= $\frac{1}{2}$ x 70^0 = 35^0

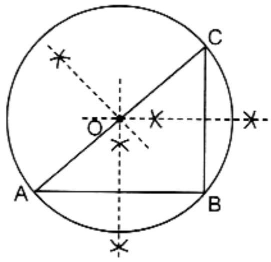
(Angle at the centre is double the angle subtended by the same arc at any point on the remaining part of the circle)

Also ∠x = ∠y (Angles in the same segment are equal)

⇒ x = 35^0

I. Short answer Type questions

1. A,B and C are three points on a circle, Prove that perpendicular bisectors of AB , BC and CA are concurrent [NCERT Exemplar]



Sol. **Given** : A, B and C are three points on the circle

To prove : Perpendicular bisector of AB, BC and CA are concurrent

Proof: i) Draw the perpendicular bisectors of AB

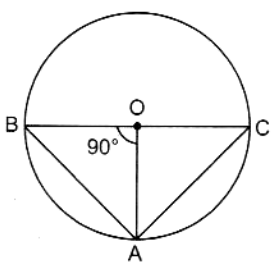
ii) Draw perpendicular bisector of BC . Both The Perpendicular Bisectors intersect at a Point 'O' This point 'O' is called the centre of the circle.

iii) Now, draw perpendicular bisector of AC. We observe that perpendicular bisector of AC also passes through the same point 0.

Hence, all the three perpendicular bisectors are concurrent, i.e. they pass through the same point.

(Reason: Three or more lines passing through the same point are called concurrent lines).

2. In the given figure AB = AC and O is the centre of the circle. If ∠BOA = 90^0, determine ∠AOC



Sol. **Given :** A circle having centre O and AB = AC.

Also, ∠AOB = 90^0

To find ∠AOC

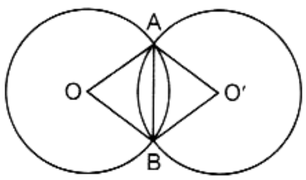
Proof : As we know that equal chords of a circle subtend equal angles at the centre of the circle.

∴ ∠AOB = ∠AOC (as AB = AC)

⇒ ∠AOC = 90^0 (as ∠AOB = 90^0)

II. Short answer Type questions

3. Two congruent circles with centres O and O' intersect at two points A and B. Then ∠AOB = ∠AO'B. Write True or false and justify your answer.



Sol : Given : Two circles with centres O and 'O' are congruent. AB is the common chord

Then ∠AOB = ∠AO'B (True)

Construction : Join OA,OB,O'A and O'B

Justification : InΔ AOB and Δ AO'B

OA = O'A (Radii of congruent circles)

OB = O'B (Radii of congruent circles)

AB = AB (Common)

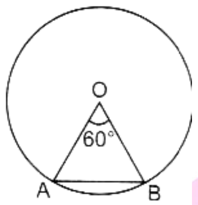
Δ AOB ≅ Δ AO'B (By SSS congruence rule)

⇒ ∠AOB = ∠AO'B (By CPCT)

Hence proved .

Therefore it is true.

4. In the given figure, chord AB subtends ∠AOB equal to 60^0 at the centre O of the circle. If OA = 5cm. Then find the length of AB.



Sol. **Given :** ∠AOB = 60^0 , OA = 5cm. Where O is the centre of the circle.

To find : AB

Proof : In ΔAOB

∠AOB = 60^0 (Given)

OA = OB (Equal radii)

∴ ∠AOB = ∠ OBA

(Angles opposite to equal sides OA and OB) ---(i)

In ΔAOB

∠OAB + ∠AOB + ∠OBA = 180^0

(Angle sum property of triangle)

60^0 + ∠OAB + ∠OAB = 180^0

(∵ ∠OAB = ∠OBA, using (i))

⇒ 2∠OAB = 180^0 - 60^0

⇒ ∠OAB = 60^0

⇒ ∠OBA = 60^0

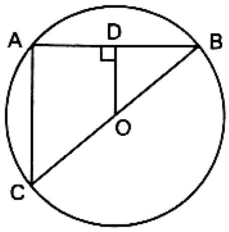
∴ ΔAOB is an equilateral triangle

Hence OA =OB = AB

⇒ AB = 5 cm (as OA = 5 cm)

III. Short answer Type questions

1. If BC is a diameter of a circle of centre O and OD is perpendicular to the chord AB of a circle. Show that CA = 2OD



Given : A circle of centre O, diameter BC and OD ⊥ chord AB.

To prove : CA = 20D

Proof : Since OD ⊥ AB.

∴ D is the mid – point of AB

(perpendicular drawn from the centre to a chord bisects the chord)

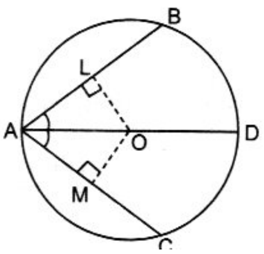
O is centre ⇒ O is the mid-point of BC

In Δ ABC, O and D are the mid points of BC and AB respectively.

∴ OD ‖ AC and OD $= \frac{1}{2}$ AC (mid-point theorem)

∴ CA = 2OD

2. If two chords of a circle are equally inclined to the diameter passing through their point of intersection, prove that the chords are equal.



Sol. Given ; Two chords AB and AC of a circle are equally inclined to diameter AOD i.e ∠DAB = ∠DAC

Construction : Draw OL ⊥ AB and OM ⊥ AC

Proof : In ΔOLA and ΔOMA

$\angle OLA = \angle OMA$ (each 90^0)

AO = AO (common)

$\angle OAL = \angle OAM$ (given)

$\Delta OLA \cong \Delta OMA$ (AAS rule)

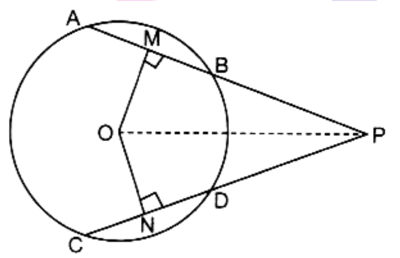
$\Rightarrow$ OL = OM (CPCT)

$\Rightarrow$ AB = AC

(chords equidistant from the centre are equal)

IV. Short answer Type questions

3. Two equal chords AB and CD of a circle when produced intersect at point p. Prove that PB = PD



Sol. **Given** : AB = CD chords AB and CD when produced meet at point P

To Prove : PB = PD

Construction : Draw OM $\perp$ AB and ON $\perp$ CD Join OP

Where O is the centre of circle

Proof : In ΔPOM and ΔPON

OM = ON (Equal chords of a circle are equidistant from the centre)

$\angle OMP = \angle ONP = 90^0$ (by construction)

OP = OP (common)

$\therefore \Delta OMP \cong \Delta ONP$ (by RHS)

$\therefore$ PM = PN (by CPCT) ----(i)

As AB = CD (given)

$\frac{1}{2}$ AB $= \frac{1}{2}$ CD

BM = DN

(Perpendicular drawn from the centre on the chord bisects the chord)

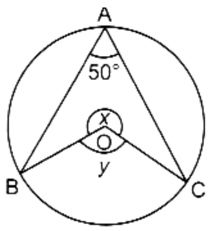
Subtracting (ii) from (i)

PM – BM = PN - DN

$\Rightarrow$ PB = PD

V. Short answer Type questions

1. Find x in the adjoining figure



Sol: Here O is the centre of the circle

$\therefore \angle BAC = \frac{1}{2}\angle Y$

(By degree measure theorem)

$\Rightarrow$ $50 = \frac{1}{2}\angle Y$

$\Rightarrow$ $\angle Y = 100^0$

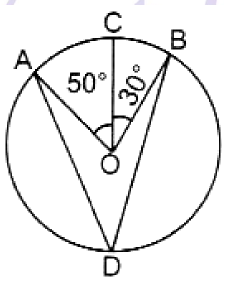
Also $\angle x + \angle y = 360^0$

(Angle at the centre of a circle)

$\Rightarrow$ $\angle x + 100^0 = 360^0$

$\Rightarrow$ $\angle x = 360^0 - 100^0 = 260^0$

2. In the given figure, O is the centre of the circle $\angle$ AOC = 50^0 and $\angle$ BOC = 30^0. Find the measure of $\angle$ ADB



Sol : Here $\angle$ AOC = 50^0 and $\angle$ BOC = 30^0

$\angle$AOB = $\angle$ AOC + $\angle$ BOC

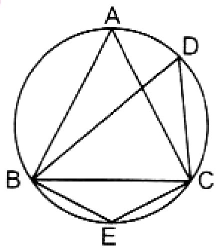
= 50^0 + 30^0 = 80^0

$\angle$AOB = = 80^0

$\angle$ ADB = $\frac{1}{2}\angle$AOB (By degree measure theorem)

$\therefore \angle$ ADB = $\frac{1}{2}$ x 80^0 = 40^0

3. In the given figure ΔABC is Equilateral. Find ∠BDC and ∠ BEC



Sol: $\angle$ BAC = 60^0

[∵ Δ ABC is Equilateral triangle)

$\therefore \angle$ BAC = $\angle$BDC

[∵ Angles in the same segment of a circle are equal)

$\Rightarrow \angle$ BDC = 60^0

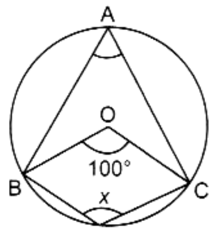
Now, □DBEC is a cycle quadrilateral

$\therefore \angle$BDC + $\angle$BEC = 180^0

[∵ Opposite angles of a cycle quadrilateral are supplementary]

60^0 + $\angle$BEC = 180^0 $\Rightarrow \angle$BEC =180^0 - 60^0 = 120^0

4. If $\angle$BOC = 100^0 then find x from the given figure.



Sol : Here O is the centre of the circle

$\therefore \angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 100^0 = 50^0$

Also $\angle x + \angle BAC = 180^0$

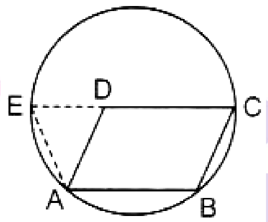
(Sum of Opposite angles of cyclic quadrilateral)

$\Rightarrow \angle x + 50^0 = 180^0 \Rightarrow x = 130^0$

V. Short answer Type questions

1. ABCD is a parallelogram. The circles through A, B and C intersect CD [produced, if necessary] at E. Prove that AE = AD

Sol. Given ABCD is a parallelogram. A circle passes through A, B and C intersect side CD produced at E



To Prove : AE = AD

Construction: Join AE

Proof: ABCD is a || gm

$\therefore$ $\angle ADC = \angle ABC$ [Opposite angels of parallelogram] .... (i)

$\angle ADC + \angle ADE = 180^0$ ....(ii)

[Angles on straight line]

Also, $\angle ABC + \angle AEC = 180^0$ .... (iii)

(Angles of cyclic quadrilateral ABCE By construction)

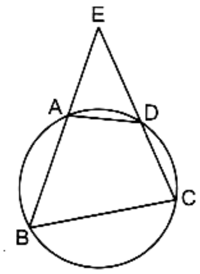
On equating (ii) and (iii)

$\angle ADC + \angle ADE = \angle ABC + \angle AEC$

$\Rightarrow$ $\angle ADE = \angle AEC$ [As $\angle ADC = \angle ABC$ opposite angles of || gm]

$\Rightarrow$ AD = AE [Sides opposite to equal angles are equal]

2. ABCD is a cyclic quadrilateral, BA and CD produced meet at E. Prove that the triangles EBC and EDA are equiangular.



Sol. **Given:** ABCD is a cyclic quadrilateral. BA and CD are produced to meet at E.

To prove : Δs EBC and EDA are equiangular

Proof : $\because$ ABCD is cyclic quadrilateral.

$\therefore \angle BAD + \angle BCD = 180^0$

[Sum of opposite angles of a cyclic quadrilateral.] ----(i)

But $\angle BAD + \angle EAD = 180^0$ [Linear pair] ------(ii)

From (i) and (ii)

$\angle BCD = \angle EAD$

Similarly, $\angle ABC = \angle EDA$

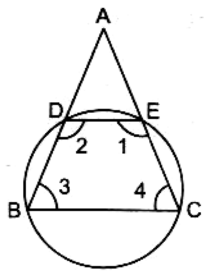
and $\angle BEC = \angle AED$

Hence, Δs EBC and EDA are equiangular

3. ABC is an isosceles triangles in which AB = AC. A circle passing through B and C intersects AB and AC at D and E respectively. Prove that BC || DE

Given : An isosceles triangle ABC in which AB = AC and a circle through B and C intersecting AB and AC at D and E respectively.

To Prove :DE || BC



Proof : In Δ ABC, AB = AC $\Rightarrow \angle 3 = \angle 4$

[Angles opposite to equal sides are equal] -----(i)

Also, DBCE is a cyclic quadrilateral

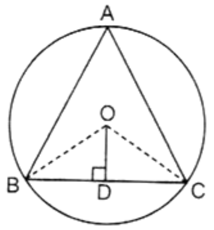
⇒ ∠2 = ∠4 = 180^0 [Opposite angles of a cyclic quadrilateral are supplementary]

⇒ ∠2 = ∠3 = 180^0 [From (i)] -------(ii)

But ∠2 = ∠3 are co-interior angles on the same side of transversal BD

∴DE || BC

4. O is the circumcentre of the triangle ABC and OD is perpendicular to BC. Prove that ∠BOD = ∠A



Construction : Join OB and OC

Proof : Here O is the centre of circle

∴ ∠BOC = 2∠A ---(i)

(By degree measure theorem)

Also, in Δ BOD and Δ COD

OB = OC (radii of circle)

OD = OD (common)

∠ ODB= ∠ ODC = 90^0 (OD ⊥ BC given)

⇒ ΔOBD ≅ ΔOCD (by RHS)

⇒ ∠ BOD = ∠COD (CPCT) -----(ii)

⇒ ∠ BOC = ∠BOD + ∠COD

= ∠BOD + ∠BOD [Using (ii)]

⇒ ∠ BOC = 2∠BOD ----(iii)

Equating (i) AND (iii)

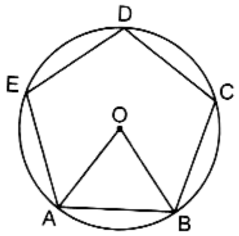
2∠A = 2∠BOD

2 ∠ BOC = ∠BOD

⇒ ∠ BOD = ∠A

I. Long answer Type questions

1. In the given figure, O is the centre of a circle and A,B,C,D and E are points on the circle such that AB = BC = CD = DE = EA. Find the value of ∠AOB.

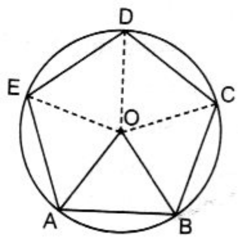


Sol : **Given :** O is centre of circle and AB = BC= CD = DE = EA

Construction : Join OC, OD, DE

To find ∠AOB.

Proof : A,B,C,Dand E are the points which lie on the circle



Also AB = BC= CD = DE = EA

All are the chords of the circle

As we know that equal chords subtend equal angle at the centre of circle.

∴ ∠AOB = ∠BOC = ∠COD

= ∠DOE = ∠AOE, ---(i)

Also ∠AOB + ∠BOC + ∠COD + ∠DOE + ∠AOE = 360^0

(sum of angles at the centre of circle)

Using (i)

∠AOB +∠AOB +∠AOB +∠AOB +∠AOB = 360^0

⇒ 5∠AOB = 360^0

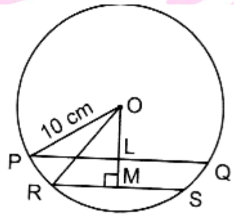
∴ ∠AOB = 72^0

2. PQ and RS are two parallel chords of a circle on the same side of centre O and radius is 10 cm. If PQ = 16 cm and RS = 12 cm, find the distance between the chords.

Sol: **Given:** A circle with centre O and two chords PQ and RS, such that PQ ∥ RS

To find : LM

Construction : Draw OM ⊥ RS which intersects PQ and L



Proof : OM ⊥ RS

∴ OL ⊥ PQ (∵PQ ∥ RS)

∴ PL = $\frac{1}{2}$ PQ and RM = $\frac{1}{2}$ RS

Now, PL = 8cm and RM = 6cm

Let LM = x cm

OP = OR = 10cm

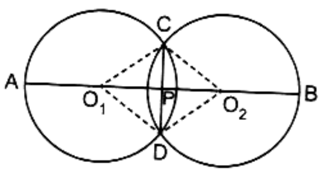
In Δ OPL, OL = $\sqrt{(10)^2 - (8)^2}$ cm = 6cm

Also, In Δ ORM, OM = $\sqrt{(10)^2 - (6)^2}$ cm = 8cm

∴ x =OM – OL = 8cm - 6 cm = 2cm

⇒ Distance between the chords = LM = 2 cm

3. O_1 and O_2 are the centres of two congruent circles intersecting each other at points C and D. The line joining their centres intersects the circles in points A and B such that AB> O_1O_2. If CD = 6 cm and AB = 12 cm determine the radius of either circle.



Sol: Let radius of each circle = r cm

AB = 12 cm

∴ O_1O_2 = 12 – 2r

Now, CD is the common chord of the two circles and O_1O_2 is the line segment that joins the centres [Radii of congruent circles]

As we know that line joining the centres of two circles is perpendicular bisector of the common chord.

∴ $O_1O_2 \perp$ CD O_1O_2 bisects CD

∴ CP = $\frac{1}{2}$ x CD = 3 cm

and O_1P = $\frac{1}{2}$ (O_1O_2) = $\frac{1}{2}(12 - 2r)$

= (6 – r) cm

Now in right Δ CPO_1

$(O_1C)^2 = (O_1P)^2 + (PC)^2$

⇒ $r^2 = (6 - r)^2 + (3)^2$

⇒ r^2 = 36 + r^2- 12r + 9

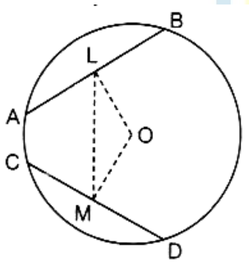
⇒ 12r = 45

⇒ r= $\frac{45}{12}$

⇒ r = 3.75cm

II. Long answer Type questions

1. Prove that the line segment joining the mid-points of two equal chords of a circle make equal angles with the chords.



Sol**: Given :** A circle C (O, r) AB and CD are two equal chords of a circle . L, M are the mid-points of AB and CD respectively.

To Prove: i) $\angle ALM = \angle CML$

ii) $\angle BLM = \angle DML$

LM, OL, OM are joined

Proof : (i) $OL \perp AB$ and $OM \perp CD$

(As the line joining the centre to the mid-point of the chord is perpendicular to the chord)

Now, $OL = OM$

[Equal chords are equidistant from the centre]

In Δ OLM $OL = OM$ [Proved above]

$\Rightarrow \angle OLM = \angle OML$

[angles opposite to equal sides are equal]-----(i)

$\angle OLA = \angle OMC$ [Each 90^0]

$\Rightarrow \angle OLA - \angle OLM = \angle OMC - \angle OML$

[$\because \angle OLA = \angle OMC = 90^0$]

$\Rightarrow \angle MLA = \angle LMC$ ----(2)

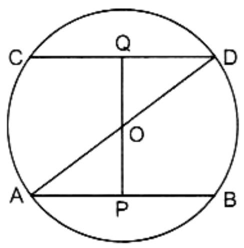
Again from (i)

$\angle OLM + OLB = \angle OML + \angle OMD$

[$\because \angle OLB = \angle OMD = 90^0$]

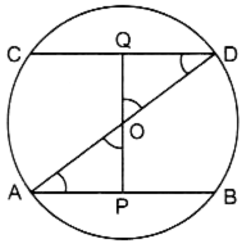
$\Rightarrow \angle MLB = \angle LMD$

2. In The given figure AB ∥CD, AD is a diameter of circle whose centre is O. Prove that AB = CD



Sol : Given : AB ∥ CD, AOD is a diameter of circle, where O is the centre of circle,

To prove : AB = CD



Proof : In Δ DOQ and Δ AOP

OD = DA (radii of angle)

∠DOQ = ∠ AOP

(Vertically opposite angle)

∠QDO = ∠PAO

[alternate angles as CD ∥ AB (given)]

⇒ ΔDOQ ≅ Δ AOP (by ASA)

⇒ OQ = OP (BY CPCT)

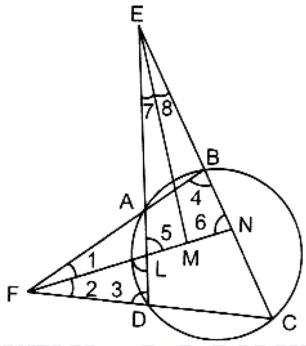
⇒ CD = AB (chords equidistant from the centre of a circle are equal)

III. Long answer Type questions

1. Prove that the angle bisectors of the angles formed by producing opposite sides of a cyclic quadrilateral [provided they are not parallel] intersect at right angle.

Sol: **Given** ABCD is a cyclic quadrilateral whose opposite sides are produced to meet at E and F.

To Prove : Bisectors of ∠E and ∠F intersect at right angle.



Proof: In ΔFEL and ΔFBN.

∠2 = ∠1 [∵ *FN is the bisector of ∠F*]

$\angle 3 = \angle 4$ [Exterior angle of cycle quadrilateral is equal to interior opposite angle

∴ Third $\angle FLD$ = Third $\angle 6$

But $\angle FLD$ = 5 [Vertically opposite angles]

∴ $\angle 5 = \angle 6$

$\Rightarrow$ EN = EL

[Sides opposite to equal angles are equal]

Now in Δ ELM and ΔENM

EL = EN [Proved above]

EM = EM [Common]

$\angle 7 = \angle 8$ [Given as EM is the bisector of $\angle E$]

∴ $\Delta ELM \cong \Delta ENM$ [SAS congruence rule]

∴ $\angle EML = \angle EMN$ [Common]

But $\angle EML + \angle EMN = 180^0$ [Linear Pair]

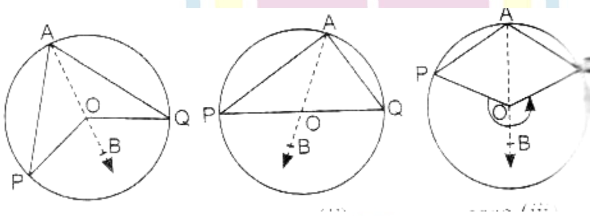
$\Rightarrow$ $\angle EML = \angle EMN = 90^0$

Hence, $EM \perp FM$.

Hence, bisectors of $\angle E$ and $\angle F$ intersect at right angle

2. Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Sol. Given an are PQ of a circle subtending angles POQ at the centre O and $\angle PAQ$ at a point A on the remaining part of the circle.



Case (i) Case (ii) Case (iii)

To prove : $\angle POQ = 2\angle PAQ$

Construction : Join AO and extends it to B

Proof: Consider three cases

Case (i) When are PQ is a minor are

Case (ii) When are PQ is a semicircle

Case (iii) When are PQ is a major are.

In all the three cases

Taking ΔAOQ

∠BOQ = ∠OAQ + ∠OQA [Exterior angle of Δ is equal to the sum of interior opposite angles]

Also OA = OQ [radii of circle]

$\Rightarrow \angle OAQ = \angle OQA$ [Angles opposite to equal sides]

$\Rightarrow \angle BOQ = \angle OAQ + \angle OAQ$

$\Rightarrow \angle BOQ = 2\angle OAQ$ .... (i)

Similarly $\angle BOP = 2\angle OAP$ .... (ii)

Adding (i) and (ii) we have

$\angle BOQ + \angle BOP = 2\angle OAQ + 2\angle OAP$

$= 2(\angle OAQ + \angle OAP)$

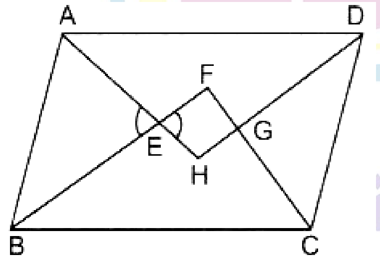
$\Rightarrow$ ∠POQ = 2∠PAQ

Specially for case (iii) we can write reflex ∠ POQ = 2 ∠PAQ

3. Prove that the quadrilateral formed [if possible] by the internal angle bisectors of any quadrilateral is cyclic.

Sol. **Given** ABCD is a quadrilateral, AH, BF, CF and DH are the angle bisectors of internal angles A,B,C and D these bisectors form a quadrilateral EFGH

To Prove: ΔEFGH is cyclic



Proof: In ΔAEB.

∠EAB + ∠ABE + ∠AEB = 180^0 [Sum of angles ofΔABC]

$\Rightarrow$ $\angle AEB = 180^0 - (\angle EAB + \angle ABE)$ ..... (i)

Also $\angle AEB = \angle FEH$ ....(ii) [Vertically opposite angle]

By equating (i) and (ii)

$\angle FEH = 180^0 - (\angle EAB + \angle ABE)$ ..... (iii)

Similarly, in ΔGDC

$\angle FGH = 180^0 - (\angle GDC + \angle GDC)$ ..... (iv)

Adding (iii) and (iv)

$\angle FEH + \angle FGH$

$= 360° - (\angle EAB + \angle ABE + \angle DDC + \angle GCD$

$= 360^0 - \frac{1}{2}(\angle BAD + \angle ABC + \angle ADC + \angle BCD)$

[As AH, BF, CF and HD are bisectors of $\angle A$, $\angle B$, $\angle C$, $\angle D$]

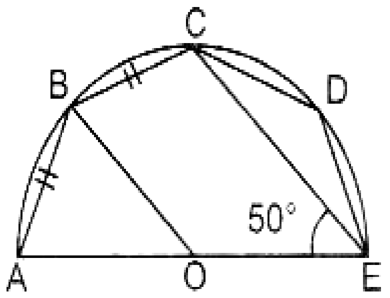
$= 360^0 - \frac{1}{2} \times 360^0$ [Sum of angles of quadrilateral, ABCD]

$\angle FEH + \angle FGH = 360^0 - 180^0 = 180^0$

$\Rightarrow$ FEHG is a cyclic quadrilateral.

[If the sum of opposite angles of quadrilateral is180^0, then it is cyclic]

4. In the given figure, O is the centre and AE is the diameter of the semicircle ABCDE. If AB = BC and $\angle$ AEC = 50^0then find (i) $\angle$CBE (ii) $\angle$CDE (iii) $\angle$AOB, Prove that BO || CE.

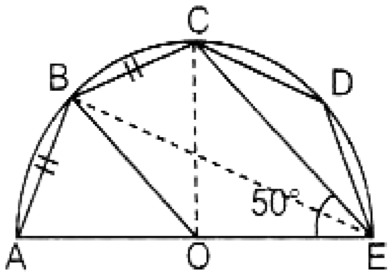


Sol. Given: O is the centre of circle and AE is the diameter of the semicircle ABCDE.

Also, AB = BC, AEC = 50^0

To find (i) $\angle$CBE (ii) $\angle$CDE (iii) $\angle$AOB, Prove that BO || CE.

Construction: Join OC and BE



Proof: ∠AOC = 2∠AEC [By degree measure theorem]

∠AOC = 2 x 50^0 = 100^0

Also ∠AOB = ∠BOC [Equal chords subtend equal angle at the centre of circle]

∴ $\angle AOB = \frac{1}{2}\angle AOC$ [Using (i)]

$\angle AOB = \frac{1}{2}100^0 = 50^0$

Now ∠AOB = ∠AEC [These are corresponding angels]

But these are corresponding angles and are equal.

∴ Line OB || CE Hence proved

(i) ∠AOC + ∠COE = 180^0 [Linear pair angles]

100^0 + ∠COE = 180^0

∠COE = 180^0 - 100^0 = 80^0

∠CBE = $\frac{1}{2}$∠COE [By degree measure theorem]

= $\frac{1}{2} \times 80^0 = 40^0$

(ii) Now, ⊡CBED is cyclic quadrilateral.

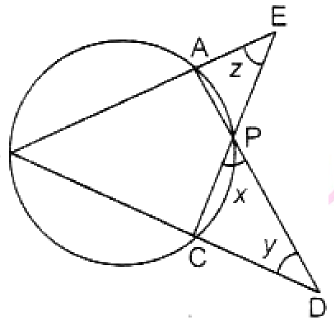
∠CBE + ∠CDE = 180^0 [Sum of opposite angles of cyclic quadrilateral

⇒ 40^0 + ∠CDE = 180^0

⇒ ∠CDE = $180^0 - 40^0 = 140^0$

(iii) ∠AOB = 50^0 (Proved above)

5. In the given figure, If y = 32^0 and z = 40^0 determine x, If y + z = 90^0, Prove that x = 45^0



Sol. Given y = 32^0 and z = 40^0

Proof: Let the line segments AD and CE cut each other at P.

Since, $\angle APE = \angle CPD$ [Vertically opposite angles]

$\angle APE = x$

Now $\angle BCP = \angle CDP + \angle CPD$ [Exterior angle]

and $\angle PAB = \angle PEA + \angle APE$ [Exterior angle]

∴ $\angle BCP = x + y$......(i)

and $\angle PAB = x + z$ ..... (ii)

Since ABCP is a cyclic quadrilateral

∴ $\angle BCP + \angle PAB = 180^0$

$\Rightarrow x + y + x + z = 180^0$

or $2x + (y + z) = 180^0$

or $2x + (40^0 + 32^0) = 180^0$ ----(iii)

or $2x = 180^0 - 72^0 = 108^0$ or $x = 54^0$

Since from (iii), we get $2x$ (y + z) = 180^0 and y + z = 90^0 (Given)

∴ $2x + 90° = 180^0$ or $2x = 90^0$

∴ $x = 45^0$