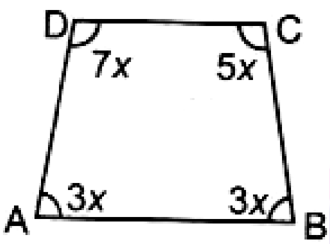
**Grade IX**

**Lesson : 8 [Quadrilaterals]**

## Objective Type Questions

### I. Multiple choice questions

1. The value of x in the given figure is




a) $10^0$  b) $20^0$  c)$30^0$  d) $40^0$

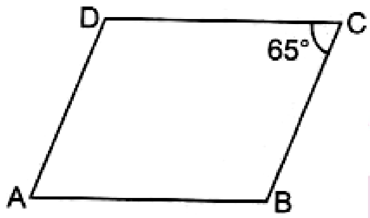
Sol : b

2. Which of the following is not a parallelogram?

a) trapezium  b) square  c) rectangle  d) rhombus

Sol : a

3. In the given figure, ABCD is a parallelogram. If $\angle C = 65^0$, then $(\angle B + \angle D)$ is equal to




a) $180^0$  b) $115^0$  c)$155^0$  d) $230^0$

**Sol** : Since ABCD is a parallelogram, so opposite angles are equal,

Thus $\angle B = \angle D$ *and* $\angle A = \angle C = 65^0$

using angle sum property of a quadrilateral, we have

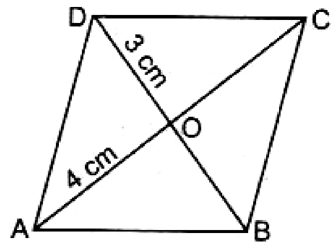
$\angle A + \angle B + \angle C + \angle D = 360^0$

$\Longrightarrow 65^0 + \angle B + 65^0 + \angle D = 360^0$

$\Longrightarrow \angle B + \angle D = 360^0 - 130^0 = 230^0$

$\therefore$ *Correct option is* $(d)$

4. In the given figure, ABCD is a rhombus, AO = 4cm and DO = 3cm. Then the perimeter of the rhombus is




a) 18cm b) 20cm c) 21cm d) 22 cm

**Sol** : Since ABCD is a rhombus, so diagonals AC and BD bisect each other right angles at O.

so$\angle$AOD = $90^0$

Now, $AD^2 = AO^2 + OD^2$ [$\because$ *Pythagora theorem*]

$\Rightarrow AD^2 = (4)^2 + (3)^2$

$\Rightarrow AD^2 = 16 + 9$

$\Rightarrow AD^2 = 25$

$\Rightarrow AD = 5\,cm$

Since all sides of a rhombus are equal, so AB = BC = CD = DA = 5cm

So, perimeter of rhombus = AB + BC + CD + DA

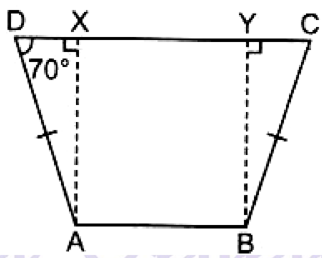
= 5 + 5 + 5 + 5 = 20cm

$\therefore$ *Correct option is* $(b)$

5. Given a trapezium ABCD, in which $AB||CD$and AD = BC. If $\angle D$= $70^0$, then $\angle C$ will be

a) $70^0$ b) $110^0$ c)$20^0$ d) *none of these*

**Sol** : Draw AX $\perp$ DC and BY $\perp$ DC




In $\triangle$AXD and $\triangle$BYC, we have

AD = BC [Given]

$\angle$AXD = $\angle$BYC [Each$90^0$]

AX = BY [ Distance between parallel sides]

So, $\triangle$AXD $\cong \triangle$BYC, [RHS congruence rule]

Thus $\angle$D = $\angle$C [CPCT]

Hence, $\angle$C = $70^0$

$\therefore$ *Correct option is* ($a$)

6.Three angles of quadrilateral are $75^0, 90^0, 75^0$, Find the fourth angle [NCERT Exemplar]

**Sol** : As we know that sum of four angles of quadrilateral is $360^0$

Let fourth angle be x.

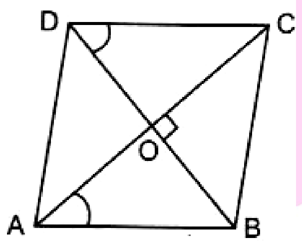
$\therefore 75^0 + 90^0 + 75^0 + \times = 360^0$

$\Rightarrow \quad \times = 360^0 - 240^0 = 120^0$

$\therefore$ *Fourth angle* $= 120^0$

7. Diagonals AC and BD parallelogram ABCD intersect at O. If $\angle$BOC = $90^0$and $\angle$BDC = $50^0$ find $\angle$OAB

**Sol** : In a parallelogram ABCD, O is point of intersection of diagonals AC and BD.




$\angle$BDC = $\angle$DBA [Alternate angles as $AC||CD$]

$\angle$BDC = $50^0$ [Given]

$\Rightarrow$ $\angle$DAB = $50^0$

We have $\angle$BOC = $\angle O$BA + $\angle OA$B

[Exterior angle is equal to sum of two interior opposite angles]

$\Rightarrow 90° = 50° + \angle$OAB

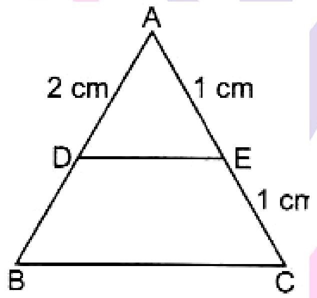
$\Rightarrow$ $\angle OA$B = $90^0 - 50^0 = 40^0$

8. Can all the angles of a quadrilateral be acute angles? Give reason for your answer. [NCERT Exemplar]

**Sol** : No, all the angles of quadrilateral cannot be acute angles. If all the angles of quadrilateral will be acute. The sum of all the four angles will be less than $360^0$ which is not possible

## II. Multiple choice questions

1. In the given figure, find BD, if $DE||BC$



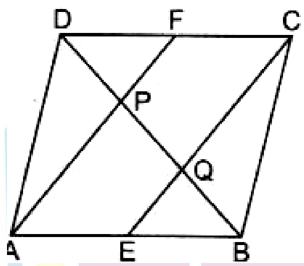

a) 2cm b) 1cm c) 3cm d) none of these

Sol : a) by the converse of the Mid-point theorem.

2. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively. AF and CE meet the diagonal BD of length 12cm at P and Q, then length of PQ is

a) 6cm b) 4 cm c) 3 cm d) 5 cm

**Sol**. In a parallelogram ABCD, we know that if E and F are the mid-points of sides AB and CD respectively, then the line segments AF and EC trisect the diagonal BD.




So DP = PQ = OB

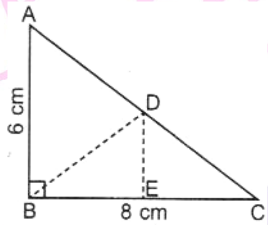
Now PQ = $\frac{1}{3}$BD = $\frac{1}{3}$x 12 = 4cm

$\therefore$ *Correct option is* $(b)$

3. A In $\triangle ABC$, right angled at B, Side AB =6cm and side BC = 8cm. D is mid-point of AC. Then length of BD is

a) 10cm b) 4 cm c) 3 cm d) 5 cm

**Sol** : By converse of mid-point theorem, we get E is mid-point of BC.




Also $\angle DEC = \angle ABC = 90^0$ [Corresponding Angle[

Now, $\triangle CED \cong \triangle BDE$ [SAS Congruence rule]

So, CD = BD [CPCT]

$\Rightarrow$ CD = BD = $\frac{1}{2}AC$ [ $\because$ D is mid-point of AC]

$\Rightarrow$ BD = $\frac{1}{2} \times 10 = 5cm$

[$\because AC = \sqrt{(AB)^2 + (BC)^2}$, Pythagoras theorem]

$\therefore$ *Correct option is* $(d)$

4. D and E are the mid-points of the sides AB and AC of $\Delta ABC$ and O is any point on side BC. O is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is [NCRT Examplar]

a) a square b) a rectangle c) a rhombus d) a parallelogram

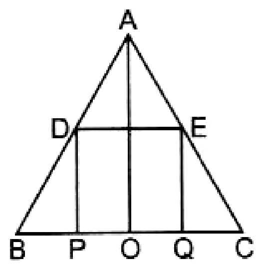
**Sol** : Using mid-point theorem, we have

$DP||EQ$

And DP = EQ = $\frac{1}{2}AO$

So, DEQP is a parallelogram,

$\therefore$ *Correct option is* $(d)$



## I. Short answer questions

1. **If one angle of a parallelogram is $36^0$ less than twice its adjacent angle, then find the angles of parallelogram** **[CBSE 2016]**

**Sol** : Let one angle of parallelogram be x.

Its adjacent angle is $(180^0 - x)$

As per question,

$x = 2\,(180^0 - x) - 36^0$

$\Rightarrow \quad x = 360^0 - 2x = 36^0$

$\Rightarrow \quad 3x = 324^0$

$\Rightarrow \quad x = \frac{324^0}{3} = 108^0$

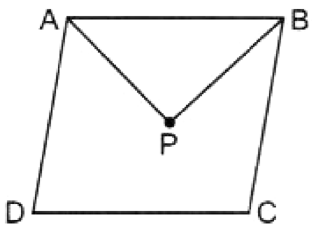
$\Rightarrow \quad Adjacent\ angle = 180^0 - 108^0 = 72^0$

Hence, the angles of parallelogram are $108^0$, $72^0$, $108^0$, $72^0$.

**2. In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angles.**

**Sol**: **Given:** ABCD is a parallelogram such that angle bisectors of adjacent angles A and B intersect at point P.

**To prove**: $\angle APB = 90^0$




**Proof** : we have

$\angle A + \angle B = 180^0$

[$AD \,||\, BC$ and $\angle A$ and $\angle B$ are consecutive interior angles]

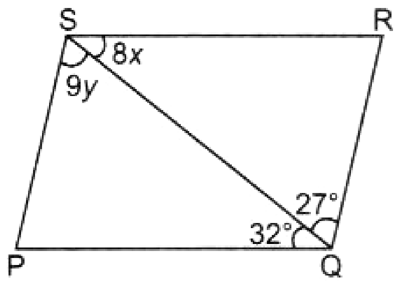
$\frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^0$

But $\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle APB = 180^0$ [Sum of angles of a triangle is $180^0$]

$\Rightarrow \quad 90^0 + \angle APB = 180^0$

$\Rightarrow \quad \angle APB = 90^0$ Hence proved

**3. In the given figure, PQRS is a parallelogram. Find the values of x and y.**




**Sol** : Here, PQRS is a parallelogram

As $PQ||RS$

$\therefore$ $8x = 32^0$ [Alternate interior angles]

As $PS||QR$

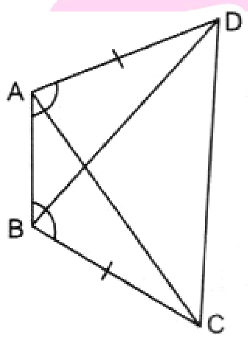
And $9y = 27^0$ [Alternate interior angles]

$\Rightarrow$ $x = \frac{32^0}{8}$ and $y = \frac{27^0}{9}$

$\Rightarrow$ $x = 4^0$ and $y = 3^0$

## II. Short answer questions

1. **In the given figure, ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA**




**Prove that : (i) $\triangle ABD \cong \triangle BAC$**

**(ii) BD = AC**

**Sol : Given** : ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA

**To prove:**

(i) $\triangle ABD \cong \triangle BAC$

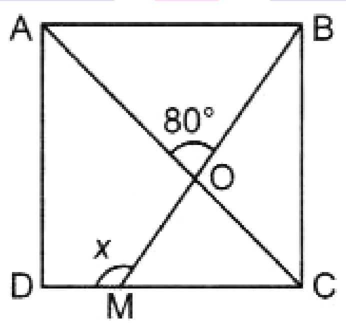
(ii) BD = AC

**Proof:** (i) In - $\triangle ABD$ and $\triangle BAC$

| | |
|---|---|
| AD = BC | [Given] |
| $\angle DAB = \angle CBA$ | [Given] |
| AB = AB | [Common] |
| $\therefore$ $\triangle ABD \cong \triangle BAC$ | [SAS congruence rule] |
| (ii) BD = AC | [CPCT] |

Hence proved

**2. In the given figure, ABCD is a square, A line BM intersects CD at M and diagonal AC at O such that $\angle AOB = 80^0$. Find the value of x.**




**Sol.** As diagonal of a square bisects the opposite angles,

$\angle BAO = \frac{1}{2}\angle BAD = \frac{1}{2} \times 90^0 = 45^0$

$\angle BAC = \angle ACD$ [Alternate interior angles]

$\therefore$ $\angle ACD = \angle BAC = 45^0$ ...... (i)

*Also* $\angle AOB = \angle MOC = 80^0$ ...... (ii) [Vertically opposite angles]

Now, $x = \angle MOC + \angle OCM$

[Exterior angle is equal to sum of two interior oppositeangles]

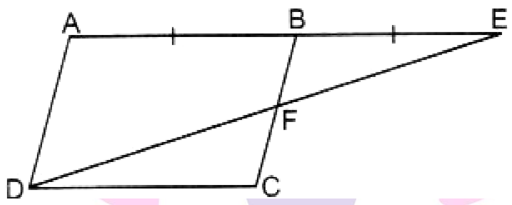
$\therefore x = 80^0 + 45^0 = 125^0$

## III. Short answer questions

**1. ABCD is a parallelogram. AB is produced to E so that BE = AB. Prove the ED bisects BC.**

**Sol : Given** : ABCD is a parallelogram. AB is produced to E such that BE = AB

**To prove** : ED bisects BC.




i.e. BF = FC

**Construction:** Join D to E which intersects BC at F.

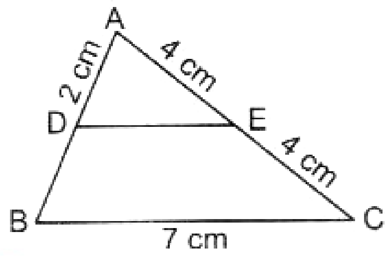
**Proof:** We have

| | | |
|---|---|---|
| | AB = DC | [Opposite sides of parallelogram] |
| But | AB = BE | [Given] |
| ∴ | BE = DC | |

In ΔBEF and ΔCDF,

| | | |
|---|---|---|
| | BE = DC | [Proved above] |
| | ∠BEF = ∠CDF | [Alternative interior angles] |
| | ∠BEF = ∠CFD | [Vertically opposite angles] |
| ∴ | ΔBEF ≅ ΔCDF, | [AAS congruence rule] |
| ∴ | BF = FC | [CPCT] |
| ∴ $ED\ bisects\ BC.$ | | Hence proved. |

**2. In the given figure, $DE||BC$ Find BD.**




**Sol** : As AE = EC = 4cm

∴ E is mid-point of AC

Also DE ∥ BC (given)

∴ By the converse of mid–point theorem, we haveD is mid-point of AB

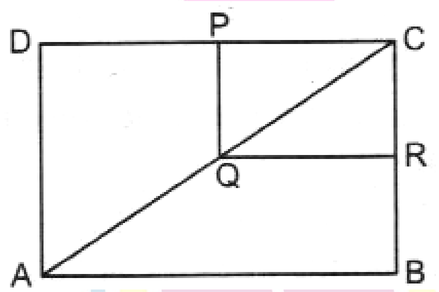
⇒ AD = BD

⇒ BD = 2cm [As given AD = 2cm]

## IV. Short answer questions

**1. In the given figure, ABCD and PQRC are rectangles and Q is the mid–point of AC. Prove that :**

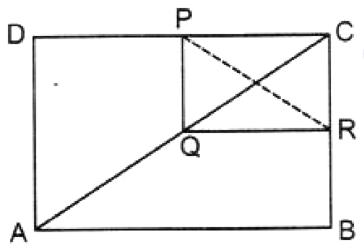
**i) DP = PC** **ii) PR = $\frac{1}{2}$ AC**




**Given :** ABCD and PQRC are two rectangle and Q is the mid-point of AC

**To prove**: i) DP = BC ii) PR = $\frac{1}{2}$ AC

**Proof** : (i) In Δ ACD,



Q is the mid-point of AC

$\angle ADC = \angle QPC = 90^0$

(Each angle of rectangle is right angle)

But these are corresponding angles.

$\Rightarrow PQ \parallel DA$

$\therefore$ P is the mid–point of CD

i.e. DP = PC

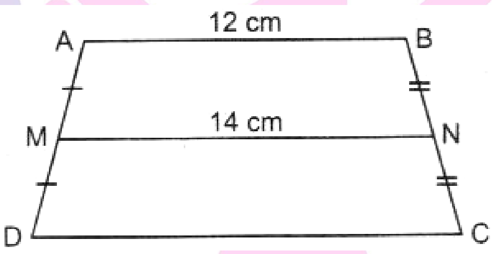
ii) We have QC = PR (Diagonals of rectangle are equal)

and QC = = $\frac{1}{2}$ AC (Given)

$\therefore$ PR = $\frac{1}{2}$ AC Hence proved

**2. ABCD is a trapezium in which AB∥DC. M and N are the mid–points of AD and BC respectively. If AB =12cm and MN =14cm, find CD. [HOTS]**

**Sol:** Here, ABCD is a trapezium in which, AB∥DC and M and N are the mid – point of AD and BC respectively.




Since the line segment joining the mid-points of non-parallel sides of trapezium is half of the sum of the lengths of its parallel sides,

$\Rightarrow MN = \frac{1}{2}(AB + CD)$

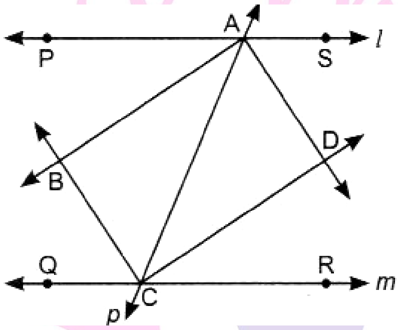
$\Rightarrow 14 = \frac{1}{2}(12 + CD)$

$\Rightarrow 28 = 12 + CD$

$\Rightarrow CD = 28 - 12 = 16$ cm

## I. Long answer questions

**1. Two parallel lines l and m are intersected by a transversal p. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.**




**Sol. Given** BA, BC, DC, DA are bisectors of ∠PAC, ∠QCA, ∠ACR and ∠SAC respectively.

**To prove** : ABCD is a rectangle.

**Proof**: We have

∠PAC = ∠ACR

[Alternate interior angles as $\boldsymbol{l}||\boldsymbol{m}$and p is transversal]

$\frac{1}{2}$∠PAC = $\frac{1}{2}$∠ACR

⟹ ∠BAC = ∠ACD

[As BA and DC are bisectors of ∠PAC and ∠ACRrespectively]

But these are alternate angles. This shows that $\boldsymbol{AB}||\boldsymbol{CD}$

Similarly, $\boldsymbol{BC}||\boldsymbol{AD}$

$\Rightarrow$ *Quadrilateral ABCD is a parallelogram* ...... (i)

Now, ∠PAC + ∠CAS = $180^0$ [Linear pari axiom]

⟹ $\frac{1}{2}$∠PAC + $\frac{1}{2}$∠CAS = $90^0$

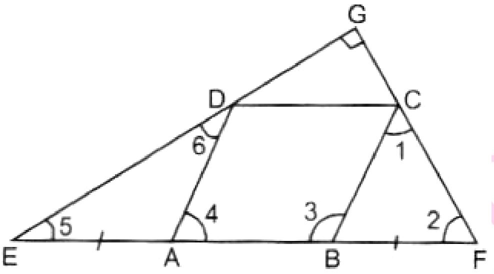
⟹ ∠BAC + ∠CAD = $90^0$

⟹ ∠BAD = $90^0$ ........ (ii)

From (i) and (ii) we can say that ABCD is a rectangle

Hence proved.

**2. ABCD is a rhombus and AB is produced to E and F such that AE = AB = BF. Prove that ED and FC are perpendicular to each other.**




**Sol. Given :** ABCD is a rhombus, AB produced to E and F such that AE = AB = BF

**Construction:** Join ED and CF and produce it to meet at G.

**To prove** ED ⊥ FC

**Proof** : AB is produced to points E and F such that

AE = AB = BF …… (i)

Also, since ABCD is a rhombus

AB = CD = BC = AD ……. (ii)

Now, in △BCF,

BC = BF [From (i) and (ii)]

⟹ $\angle 1 = \angle 2$

$\angle 3 = \angle 1 + \angle 2$ [Exterior angle ]

$\angle 3 = 2\angle 2$ …… (iii)

Similarly, AE= AD

$\angle 5 = \angle 6$

⟹ $\angle 4 = \angle 5 + \angle 6 = 2\angle 5$ …….(iv)

Adding (iii) and (iv), we get

$\angle 4 + \angle 3 = 2\angle 5 + 2\angle 2$

⟹ $180^0 = 2(\angle 5 + \angle 2)$

[ ∵ $\angle 4$ *and* $\angle 3$ *are consecutive interior angles*]

⟹ $\angle 5 + \angle 2 = 90^0$

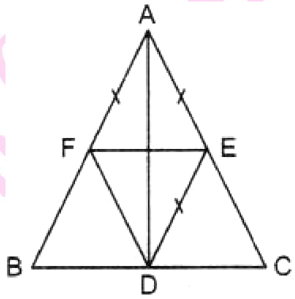
∴ EG ⊥FC, Now in ΔEGF

$\angle 5 + \angle 2 + \angle EGF = 180^0$

⟹ $\angle EGF = 90^0$ Hence proved.

# I. Long answer questions

**1. In $\Delta ABC$ is isosceles with AB = AC, D, E and F are the mid-point of sides BC, CA and AB respectively. Show that the line segment AD is perpendicular to the line segment EF and is bisected by it.**




**Sol : Given** $\Delta ABC$ is isosceles with AB = AC, D, E and F are the mid-point of BC, CA and AB respectively.

**To Prove :** AD ⊥ EF and is bisected by it.

**Construction:** Join D, E and F and AD

**Proof**: we have

DE ∥ AB and DE = $\frac{1}{2} AB$ .......(i)

And DF∥ AC and DF = $\frac{1}{2} AC$ .......(ii)

(Line segment joining mid-points of two sides of a triangle is parallel to the third side and is half of it.)

AB = AC ......(iii)

∴ AF = $\frac{1}{2}$AB, AE = $\frac{1}{2}$AC ...... (iv)

From (i), (ii), (iii), and (iv), we get

DE = DF = AF = AE

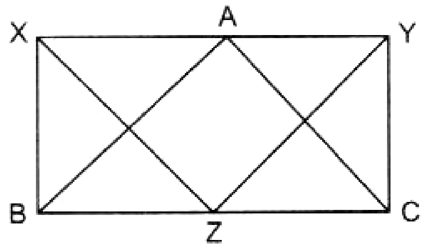
And also, DF ∥ AE and DE ∥ AF

⟹ DEAF is a rhombus.

Since diagonals of a rhombus bisect each other at right angles,

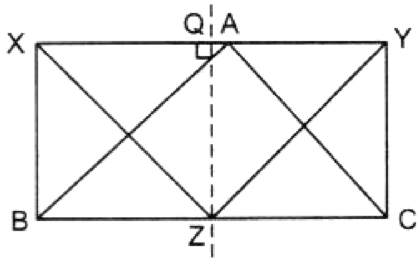
∴ AD ⊥ EF and is bisected by it.

**2. In the given figure, BX and CY are perpendicular to a line through the vertex A of $\Delta ABC$ and Zis the mid-point of BC. Prove that XZ = YZ [HOTS, CBSE 2015]**




**Sol : Given** BX and CY are perpendiculars to a line XAY

**To prove:** XZ = YZ




**Construction :** Draw $ZQ \perp XY$

**Proof**: We have

$BX \perp XY$ [Given]

And $CY \perp XY$ [Given]

Also, $ZQ \perp XY$ [By construction]

$BX \parallel ZQ \parallel CY$

(Perpendiculars on same line are parallel to each other)

Now, As $BX \parallel ZQ \parallel CY$ and Z is mid-point of BC

By mid-point theorem, we have Q is mid-point of XY.

In $\Delta$**X**QZ and $\Delta YQZ$,

XQ = QY [Q is mid-point of XY]

$\angle XQZ = \angle YQZ = 90^0$ [By construction]

QZ = QZ [Common]

$\therefore$ $\Delta XQZ \cong \Delta YQZ$ [SAS congruence rule]

$\Rightarrow$ XZ = YZ [CPCT]

Hence proved.