Name : ____________________________

Grade : VIII

Subject : Mathematics

Chapter :14. Factorisation

Objective Type Questions **1 Marks.**

I. Multiple choice questions

1. Common factor of 17abc, 34ab2, 51a2b is

a. 17abc b. 17ab c. 17ac d. 14a2b2c

2. Which of the following is a factor of 6xy – 4y + 6 – 9x?

a. 2x + y b. x – y c. 2x – 3 d. 3x – 2

3. Which of the following is a factor of y2 - 7y + 12?

a. 2y + 3 b. y + 3 c. y = 3 d. 2y – 2

4. Which of the following is factor of m4 – 256?

a. m + 4 b. m2 + 4 c. m2 – 4 d. 2y – 2

5. Which of the following is a factor of z2 – 4z – 12?

a. z + 6 b. z – 6 c. z2 – 12 d. z + 2

6. On dividing 57p2 qr by 114 pq, we get:

a. $\frac{1}{4}pr$ b. $\frac{3}{4}pr$ c. $\frac{1}{2}pr$ d. 2 pr

7. Factorizing 10xy (16x2 – 9y2), we get:

a. 10xy (4x + 3y) (4x – y) b. 10xy (4x + 3y) (4x – 3y)

c. (4x + 3y) (4x – 3y) d. 10xy (3x + 4y) 3x – 4y)

8. On dividing p(4p2 – 16) by 4p(p – 2), 23 get

a. 2p + 4 b. 2p – 4 c. p + 2 d. p – 2

9. (p3q6 – p6q3) ÷ p3 q3 equal to

a. p3qe b. –p3q3 c. p3 – q3 d. q3 – p3

10. The common factor of 3ab and 2cd is

a. 1 b. -1 c. a d. c

11. An irreducible factor of 24x2y2 is

a. x2 b. y2 c. x d. 24x

12. Number of term in factors of (a + b)2 is

a. 4 b. 3 c. 2 d. 1

13. Which one of the following is not a factor of x3 + 2x2 + x?

a. x   b. x + 1   c. x + 2   d. x(x + 1)

14. The factorised form of 3x – 24 is

a. $3x \times 24$ b. $3\ (x-8)$ c. $24\ (x-3)$ d. $3\ (x-12)$

15. Factorised form of 23xy – 46x + 54y – 108 is

a. $23x+54)\ (y-2)$ b. $(23x+54y)(y-2)$

c. $(23xy+54y)(-46x-108)$ d. $(23x+54)(y+2)$

16. The factors of x2 – 4 are

a. $(x-2),(x-2)$ b. $(x+2),(x-2)$

c. $(x+2),(x+2)$ d. $(x-4),(x-4)$

17. Factorised form of p2 – 17p – 38 is

a. $(p-19)(p+2)$ b. $(p-19)(p-2)$

c. $(p+19)(p+2)$ d. $(p+19)(p-2)$

18. Factorised form of r2 – 10r + 21 is

a. $r-1)\ (r-4)$ b. $(r-7)(4-3)$

c. $(r-7)(\ +3)$ d. $(4+7\ )(r+3)$

19. The value of (2x2 + 4) ÷ 2 is:

a. 2x2 + 2   b. x2 + 2   c. x2 + 4   d. 2x2 + 4

20. The value of (a + b)2 – (a – b)2 is

a. 4ab   b. -4ab   c. 2a2 + 2b2   d. 2a2 – 2b2

1. b	2. d	3. c	4. a	5. b	6. c	7. a	8. c	9. d	10. a
11. c	12. c	13. c	4. b	15. a	16. b	17. a	18. b	19. b	20. a

II. Multiple choice questions

1. The irreducible factorisation of $3a^3$+ 6a is

a) 3a (a^2+2) b) $3(a^3+2)$ c) a ($3a^2+6$)   d) 3 x a x a x a +2 x 3 x a

2. The value of $(3x^3+9x^2+27x) \div 3x$ is

a) $x^2+9+27x$ b) $3x^3+3x^2+27x$ c)$3x^3+9x^2+9$ d) x^2+3x+9

3. Common factor of 17abc, $34ab^2$, 51 a^2b is

a) 17 abc   b) 17 ab   c) 17 ac   d) $17a^2b^2c^2$

4. Factorised form of r^2-10r+21 is

a) (r-1) (r-4)   b) (r-7) (r-3)   c) (r-7) (r+3)   d) (r+7) (r+3)

5. The value of $(-27x^2y) \div (-9xy)$ is

a) 3xy   b) -3xy   c) -3x   d) 3x

6. The common factor of 3ab and 2cd is

a) 1   b) -1   c) a   d) c

7. Number of factors of $(a + b)^2$ is

a) 4 b) 3 c) 2 d) 1

8. O dividing p $(4p^2-16)$ by 4p (p-2), by we get

a) 2p +4 b) 2p – 4 c) p + 2 d) p – 2

9. The factors of $x^2 - 4$ are

a) (x – 2), (x – 2) b) (x + 2), (x – 2) c) (x + 2), (x + 2) d) (x – 4), (x – 4)

10.On dividing $57p^2qr$ by 114pq we get

a)$\frac{1}{4}\ pr$ b)$\frac{3}{4}\ pr$ c) $\frac{1}{2}\ pr$ d) 2pr

1. a	2. d	3. b	4. b	5. d	6. a	7. c	8. c	9. b	10. c

I. Fill in the blanks

1. (a – b) ______________ = a2 – 2ab + b2
2. a2 – b2 = (a + b) ______________ .
3. (a – b)2 + ______________ = a2 – b2.
4. (a + b)2 – 2ab = ______________ + ______________.
5. (x – a) (x + b) = x2 + (a + b) x + ___________.

1. (a – b)	2. (a – b)	3. 2ab – 2b2	4. a2 + b2	5. ab

I. True or False

1. The factors of a2 – 2ab + b2 are (a + b) and (a + b).
2. h is a factor of 2p (h + r)
3. Some of the factors of $\frac{n^2}{2} + \frac{n}{2}\ are \frac{1}{2}, n\ and\ (n + 1)$.
4. An equation is true for all values of its variables.
5. x2 = (a – b)x + ab = (a + b) (x + ab)

1. False	2. False	3. True	4. False	5. False

I. Very short answer type questions.

1. Write the greatest common factor in each of the following.

a. 13x2y, 169xy b. 3x2y, 18xy2, - 6xy

Sol. a. ∴ $13x^2y = 13 \times x \times x \times y$

$169xy \times 13 \times x \times y$

∴ The common factor = $13 \times x \times y = 13xy$

b. ∴ $3x^2y = 3 \times x \times x \times y$

$$18xy^2 = 2 \times 3 \times 3 \times x \times y \times y$$

$$-6xy = -2 \times 3 \times x \times y$$

$\therefore$ The common factor $= 3 \times x \times y = 3xy$

2. Factorize the following expressions.

a. $\mathbf{6ab + 12bc}$ b. $\mathbf{ax^3 - bx^2 + cx}$ [NCERT Exemplar]

Sol. a. $6ab + 12bc$

Taking common factor in the above equation.

$$= 6b(a + 2c)$$

b. $ax^3 - bx^2 + cx$

Taking common factor in the above equation.

3. Factorise: $\mathbf{49x^2 - 36y^2}$ [NCERT Exemplar]

Sol. $49x^2 - 36y^2 = (7x)^2 - (6y)^2$

$= (7x + 6y)(7x - 6y)$

$[using\ a^2 - b^2 = (a + b)]$

4. Factorise: $\mathbf{25ax^2 - 25a}$ [NCERT Exemplar]

Sol. $25ax^2 - 25a = 25a(x^2 - 1)$

$= 25a(x + 1)(x - 1)$

$[Using\ a^2 - b^2 = (a + b)(a - b)]$

5. Factorise: $\mathbf{\frac{x^2}{9} - \frac{y^2}{25}}$

Sol. $\frac{x^2}{9} - \frac{y^2}{25} = \left(\frac{x}{3}\right)^2 - \left(\frac{y}{5}\right)^2$

$= \left(\frac{x}{3} + \frac{y}{5}\right)\left(\frac{x}{3} - \frac{y}{5}\right)$

6. Factorise: $\mathbf{a^3 + a^2 + a + 1}$ [NCERT Exemplar]

Sol. $a^2 + a^2 + a + 1 = a^2 (a + 1) + 1 (a + 1)$

$= (a + 1) (a^2 + 1)$

7. Factorise: $\mathbf{lx + my + mx + ly}$ [NCERT Exemplar]

Sol. $lx + my + mx + ly = lx + mx + my + ly$

$= x(l + m) + y(l + m)$

$= (l + m)(x + y)$

8. Factorise: $l^2 m^2 n - lm^2n^2 - l^2 mn^2$ [NCERT Exemplar]

Sol. $l^2 m^2 n - lm^2n^2 - l^2 mn^2 = lmn [lm - mn - ln]$

9. Factorise: $3a^2b3 - 27a^4b$ [NCERT Exemplar]

Sol. $3a^2b3 - 27a^4b = 3a2n (b2 - 9a2)$

$= 3a2^b [(b)2 - (3a)^2]$

$= 3a^2b (b + 3a) (b - 3a)$

10. Factorise: $9x^2 - 1$

Sol. $9x^2 - 1 = (3x)^2 - (1)^2$

$= (3x + 1) (3x - 1)$

II. Very short answer type questions.

1. Factorise x^2-9

Solution:

$x^2-9 = x^2-3^2$

We know identity $a^2-b^2 = (a+b)(a-b)$ here a = x, b = 3

$x^2-3^2 = (x+3)(x-3)$

2. Factorise $x^3y^2 + x^2y^3 - xy^4 + xy$.

Solution:

$x^3y^2 + x^2y^3 - xy^4 + xy$

Taking common factors in the above expression, we get

$= xy [x^2y + xy^2 - y^3 + 1]$

3. Factorise $p^2 - 2p + 1$

Solution:

$p^2 - 2p + 1 = (p)^2 - 2 \times p \times 1 + (1)^2$

Using identify $(a-b)^2 = a^2+b^2-2ab$

Here a= p, b = 1

$p^2 - 2p + 1 = (p-1)^2 = (p-1)(p-1)$

4. Factorise $a^2x^2 + 2ax + 1$

Solution:

$a^2x^2+2ax+1 = (ax)^2+2(ax) \times 1 + (1)^2$

Using identity $(a+b)^2 = a^2+b^2$-2ab

Here a = ax, b=1

$a^2x^2+2ax+1 = (ax+1)^2 = (ax+1)(ax+1)$

5. Divide: 7x2y2z2 ÷ 14xyz

Solution: $\frac{7x^2y^2z^2}{14xyz} = \frac{7 \times x \times x \times y \times y \times z \times z}{14 \times x \times y \times z} = \frac{1}{2}xyz$

I. Short answer type questions.

1. Factorise the following.

a. $\mathbf{4x^2 - 20x + 25}$ b. $\mathbf{x^4 - 256}$ [NCERT Exemplar]

Sol. a. $4x^2 - 20x + 25 = (2x)^2 - 2 \times 2x \times 5 + (5)^2$

$= (2x-5)^2$

$[Since, a^2 - 2ab + b^2 = (a-b)^2]$

$(2x-5)(2x-5)$

b. $x^4 - 256 = (x^2)^2 - (16)^2$

$= (x^2+16)(x^2-16)$

$[using\ a^2-b^2=(a+b)(a-b\ )]$

$= (x^2+16)(x^2-4^2)$

$= (x^2+16)(x+4)(x-4)$

2. Factorise the following.

a. $\mathbf{x^2+9x+20}$ b. $\mathbf{p^2-13p-30}$ [NCERT Exemplar]

Sol. a. $x^2+9x+20 = x^2+(5+4)x = 20$

$= x^2+5x+4x+20$

$= x(x+5)+4(x+5)$

$= (x+5)(x+4)$

b. $p^2-13p-30 = p^2-(15-2)p-30$

$= p^2-15p+2p-30$

$= p(p-15)+2(p-15)$

$= (p-15)(p+2)$

3. Factorise the following.

a. $\frac{x^2}{4}+2x+4$ b. $16x^2+40x+25$ [NCERT Exemplar]

Sol. a. $\frac{x^2}{4}+2x+4 = \frac{1}{4}[x^2+8x+16]$

$= \frac{1}{4}[x^2+4x+4x+16]$

$= \frac{1}{4}[x(x+4)+4\ (x+4)]$

$= \frac{1}{4}[(x+4)(x+4)]$

$= \frac{1}{4}(x+4)^2$

b. $16x^2+40x+25 = 16x^2+(20+20)x+25$

$= 16x^2+20x+20x+25$

$= 4x\ (4x+5)+5(4x+5)$

$= (4x+5)(4x+5)$

$= (4x+5)^2$

4. Factorise the following.

a. $\mathbf{\frac{x^3y}{9}-\frac{xy^3}{16}}$ b. $\mathbf{x-1}$ [NCERT Exemplar]

Sol. a. $\frac{x^3y}{9}-\frac{xy^3}{16} = xy\left[\frac{x^2}{9}-\frac{y^2}{16}\right]$

$= xy\left[\left(\frac{x}{3}\right)^2-\left(\frac{y}{4}\right)^2\right]$

$= xy\left[\left(\frac{x}{3}+\frac{y}{4}\right)\ \left(\frac{x}{3}-\frac{y}{4}\right)\ \right]$

$[\therefore a^2-b^2(a+b)(a-b)]$

$= xy\left(\frac{x}{3}+\frac{y}{4}\right)\left(\frac{x}{3}-\frac{y}{4}\right)$

b. $x - 1 = (x^2)^2 - (1)^2$

$= (x^2 + 1)(x^2 - 1)$

$= (x^2 + 1)(x + 1)(x - 1)$

$[\therefore a^2 - b^2 = (a + b)(a - b)]$

5. Carry out the following diversion.

a. $51x^3y^2z \div 17xyz$ **b. $76x^3yz^3 \div 19x^2y^2$** **[NCERT Exemplar]**

Sol. a. $51x^3y^2z \div 17xyz = \frac{51x^3y^3z}{17xyz}$

$= \frac{3 \times 17 \times x \times x \times x \times y \times y \times z}{17 \times x \times y \times z}$

$= 3 \times x \times x \times y = 3x^2y$

b. $76x^3yz^3 \div 19x^2y^2 = \frac{76x^3yz^3}{19x^2yz}$

$= \frac{4 \times 19 \times x \times x \times x \times y \times z \times z \times}{19 \times x \times x \times y \times y}$

$= \frac{4 \times x \times z \times z \times z}{y} = \frac{4xz^3}{y}$

6. Factorise : $16x^4 - 625y^4$ **[NCERT Exemplar]**

Sol. $16x^4 - 625y^4 = [4x^2]2 - [25y^2]^2$

$= [4x^2 + 25y^2][4x^2 - 25y^2]$

$[\therefore a^2 - b^2 = (a + b)(a - b)]$

$= [4x^2 + 25y^2][(2x)^2 - (5y)^2]$

$= [4x^2 + 25y^2][(2x + 5y)(2x - 5y)]$

$= (4x^2 + 25y^2)(2x + 5y)(2x - 5y)$

7. Factorise: $x^4 - y^4 + x^2 - y^2$ **[NCERT Exemplar]**

Sol. $x^2 - y^4 + x^2 - y^2 = (x^2)^2 - y^2)^2 + x^2 - y^2$

$[\therefore a^2 - b^2 = (a + b)(a - b)]$

$= (x^2 + y^2)(x^2 - y^2) + (x^2 - y^2)$

$= (x^2 - y^2)(x2 + y^2 + 1)$

$= (x + y)(x - y)(x^2 + y2 + 1)$

$[\therefore a^2 + b^2 = (a + b)(a - b)$

8. Factorise the following.

a. $\frac{1}{36}a^2b^2 - \frac{16}{19}b^2c^2$

b. $49ax^2 - 36y^2$ **[NCERT Exemplar]**

a. $\frac{1}{36}a^2b^2 - \frac{16}{19}b^2c^2 = b^2\left[\frac{1}{36}a^2 - \frac{16}{49}c^2\right]$

$= b^2\left[\left(\frac{a}{6}\right)^2 - \left(\frac{4}{7}c\right)^2\right]$

$= b^2\left[\frac{a}{6} + \frac{4}{7}c\right]\left[\frac{a}{6} - \frac{4}{7}c\right]$

b. $49x^2 - 36y^2 = (7x)^2 - (6y)^2$

$= (7x + 6y)(7x - 6y)$

[$\therefore a^2 - b^2 = (a + b) a - b)$]

9. Factorise: $(x + y)^4 - (x - y)^4$ **[NCERT Exemplar]**

Sol. $(x + y)^4 - (x - y)^4 = [(x + y)^2]^2 - [(x - y)^2]^2$

$= [(x + y)^2 + (x - y)^2] [(x + y)^2 - (x - y)^2]$

$= [x2 + 2xy + y^2 + x^2 - 2xy + y^2] [(x +y) + (x - y)]$

$[(x + y) - (x-y)]$

$= [2x^2 + 2y^2] [2x \times 2y]$

$= 2(x^2 + y^2) [4xy]$

$= 8xy (x^2 + y^2)$

10. Factorise the following.

a. $18 + 11x + x^2$

b. $y2 - 2y - 15$ **[NCERT Exemplar]**

Sol. a. $18 + 11x + x^2 = x^2 + 11x + 18$

$= x^2 + (9 + 2) x + 18$

$= x^2 + 9x + 2x + 18$

$= x (x + 9) + 2 (x + 9)$

$= (x + 9) (x + 2)$

b. $y^2 - 2y - 15 = y^2 - (5 - 3) y - 15$

$= y^2 - 5y + 3y - 15$

$= y(y - 5) + 3(y - 5)$

$= (y - 5) (y + 3)$

II. Short answer type questions.

1. Factorise $2ax^2 + 4axy +3bx^2 +2ay^2 +6bxy +3by^2$.

Solution:

$2ax^2 + 4axy +3bx^2 +2ay^2 +6bxy +3by^2$

Grouping $2ax^2 + 4axy +3bx^2 +2ay^2$, we get

$2a [x^2 + 2xy +y^2]$ …….(i)

Taking common factor of the terms $3bx^2 +6bxy + 3by^2$, we get

$3b[x^2 +2xy +y^2]$ ……..(ii)

Putting (i) and 9ii) together

$= 2a [x^2 +2xy +y^2] +3b [x^2+2xy +y^2]$

$= [2a +3b] [x+y]^2$

2. Factorize: (i) $\frac{x^2}{4} + 2x + 4$ (ii) $2x^3+24x^2+72x$

Solution: $(i)\ \frac{x^2}{4} + 2x + 4$

Taking ¼ as common factor in above equation, we get

$\frac{x^2}{4} + 2x + 4$ = ¼ [x^2 +8x +16]

Now, x2+8x +16 =x^2 + 4x + 4x + 16 [as 4 x 4 =16 and 4 + 4 =8]

=x (x + 4) + 4 (x + 4) = (x+4) (x+4)

Therefore, $\frac{x^2}{4} + 2x + 4$ = ¼ [(x+4) (x +4)] = ¼ $(x+4)^2$

(ii) $2x^3+24x^2+72x$

Taking 2x common from all terms of expression, we get

$2x^3+24x^2+72x = 2x (x^2+12x +36)$

Now, $x^2+12x +36 = x^2 +6x+6x+36$

=x(x+6) 6(x+6) = (x+6) (x+6)

There fore , $2x^3+24x^2+72x = 2x [(x+6) (x+6)] =2x (x+6)^2$

3. Factorise (i) $9y^2 - 4xy + \frac{4x^2}{9}$ (ii) $9x^2$ -12x +4

Solution: $9y^2 - 4xy + \frac{4x^2}{9} \quad = (3y)^2 - 2 \times 3y \times \frac{2x}{3} + \left(\frac{2x}{3}\right)^2$

Using identity $(a-b)^2 = a^2 + b^2 + 2ab$

Here a = 3y, b=$\frac{2}{3}$x

$\therefore 9y^2 - 4xy + \frac{4x^2}{9} \quad = \left(3y - \frac{2}{3}x\right)^2$

(ii) $9x^2$ -12x +4

Using identity $(a-b)^2 = a^2 + b^2 + 2ab$

Here a = 3x, b= 2

$(3x)^2$ -3 (3x) (2) + $(2)^2$ = $(3x-2)^2$ = (3x-2) (3x-2)

4. Factorise: (i) x^2+4x-77 (ii) y^2+7y+12

Solution:

(i) X^2+4x-77

We know 77= 11 x 7 and 11= 4.Therefore,

X^2 + 4x - 77 = x^2 + 11x - 7x - 77 = x (x+11) - 7(x+11) = (x-7) (x+11)

(ii) Y^2+7y+12

We know 12= 4 x 3 and 4 +3 =7 therefore,

Y^2 + 7y + 12 = y^2 + 4y + 3y + 12 = y (y+4) + 3(y+4) = (y+3) (y+4)

5. (i)$\frac{4x^2}{9} - \frac{9y^2}{16}$ (ii) x4 -1

Solution:

(i) $\frac{4x^2}{9} - \frac{9y^2}{16} = \left(\frac{2x}{3}\right)^2 - \left(\frac{3y}{4}\right)^2$

Using identity $(a-b)^2 = a^2 + b^2 + 2ab$, where $a = \frac{2x}{3}$ $b = \frac{3y}{4}$

$$\left(\frac{2x}{3}\right)^2 - \left(\frac{3y}{4}\right)^2 = \left(\frac{2x}{3} + \frac{3y}{4}\right)\left(\frac{2x}{3} - \frac{3y}{4}\right)$$

(ii) $X^4 - 1 = (x^2)^2 - 1^2$

Using identity $(a-b)^2 = a^2 + b^2 + 2ab$, where $a = x^2$ $b = 1$

$(x^2)^2 - 1^2 = (x^2+1)(x^2-1)$

Again using the same identify in term (x^2-1) we get,

$X^4 - 1 = (x^2+1)(x+1)(x-1)$

6. Factorise $(x+y)^4 - (x-y)^4$.

Solution:

$(x+y)^4 - (x-y)^4$.

Using identify $a^2 - b^2 = (a+b)(a-b)$

Here $a = (x+y)^2$, $b = (x-y)^2$

$((x+y)^2)^2 - (x-y)^2)^2 = [(x+y)^2 + (x-y)^2][(x+y)^2 - (x-y)^2]$

Using identify $(a+b)^2 = a^2+b^2+2ab$ and $(a-b)^2 = a^2+b^2-2ab$

$= [x^2+y^2+2xy + x^2+y^2-2xy][x^2+y^2+2xy - (x^2+y^2-2xy)]$

$= [2x^2+2y^2][x^2+y^2+2xy-x^2-y^2+2xy]$

$= [2x^2+2y^2][4xy]$

$= 8xy[x^2 + y^2]$

7. If p+q and pq =22, then find p^2+q^2.

Solution:

Given $p+q=12 \rightarrow (p+q)^2 = 12^2$

- ➔ $P^2 + 2pq + q^2 = 144$
- ➔ $P^2 + q^2 = 144 - 2pq$
- ➔ $P^2 + q^2 = 144 - 2 \times 22 = 144 - 44 = 100$

I. Long answer type questions.

1. Factorise and divide the following.

a. $(x^2 - 22x + 117) \div (x - 13)$

b. $(9x^2 - 4) \div (3x + 2)$ [NCERT Exemplar]

Sol. a. $(x^2 - 22x + 117) \div (x - 13)$

$\therefore$ $x^2 - 22x + 117 = x^2 - 13 + 9) x + 117$

$= x^2 - 13x - 9x + 117$

$= x (x - 13) - 9 (x - 13)$

$= (x - 13) (x - 9)$

$\therefore$ $\frac{x^2-22x+117}{(x-13)} = \frac{(x-13)(x-9)}{(x-13)} = x - 9$

b. $(9x^2 - 4) \div (3x + 2)$

$\because$ $9x^2 - 4 = (3x)^2 - (2)^2$

$= (3x + 2) (3x - 2)$

$\therefore$ $\frac{9x-4}{(3x+2)} = \frac{(3x+2)\,(\,(3x-2)}{(3x+2)} = (3x - 2)$

2. Factorise and divide the following.

a. (2x3 – 12x2 + 16x) ÷ (x – 2) (x – 4)

b. (3x4 – 625) ÷ (3x2 – 75)

Sol. a. (2x3 – 12x2 + 16x) ÷ (x – 2) (x – 4)

$\because$ 2x3 – 12x2 + 16x = 2x (x2 – 6x + 8)

= 2x (x2 – 4x – 2x + 18)

= 2x [x (x – 4) – 2 (x – 4)]

= 2x [x – 4) (x – 2)]

= 2x (x – 2) (x – 4)

$\therefore$ $\frac{2x^2-12x^2+16x}{(x-2)(x-4)} = \frac{2x\,(x-2)(x-4)}{(x-2)(x-4)} = 2x$

b. $(3x^4 - 625 \div (3x^2 - 75)$

$\because$ $3x^4 - 625 = 3 (x^4 - 625)$

$= 3[(x^2)^2 - (25)^2]$

$= 3 [(x^2 + 25) (x^2 - 25)]$

$= 3 [(x^2 + 25) (x^2 - 52)]$

$= 3 [(x^2 + 25) (x + 5) (x - 5)]$

and $3x^2 - 75 = 3 (x^2 - 25)$

$= 3 [(x)^2 - (5)^2]$

$= 3 (x + 5) (x - 5)$

$\therefore \quad \frac{3x^4-625}{3x^2-75} = \frac{3(x^2+25)(x+5)(x-5)}{3(x+5)(x-5)} = (x^2+25)$

3. Factorise the following.

a. (a) $a^3 - 4a^2 + 12 - 3a$ b. $4x^2 - 20x + 25$ [NCERT Exemplar]

Sol. (a) $a^3 - 4a^2 + 12 - 3a = a^2(a-4) - 3a + 12$

$= a^2(a-4) - 3(a-4)$

$= (a-4)(a^2-3)$

b. $4x^2 - 20x + 25 = (2x)^2 - 2\times 2x \times 5 + (5)^2$

[Since, $a^2 - 2ab + b^2 = (a-b)^2$]

$= (2x-5)(2x-5)$

4. Factorise the following:

(a) x^4-y^4 (b) $16x^4 - 81$ [NCERT Exemplar]

Sol. (a) $x^4-y^4 = (x^2)^2 - (y^2)^2$

$= (x^2+y^2)(x^2 y^2)$

$= (x^2+y^2)(x+y)(x-y)$

(b) $16x^4 - 81 = (4x^2)^2 - (9)^2$

$= (4x^2+9)(4x^2-9)$

$= (4x^2+9)[(2x)^2-(3)^2]$

$= (4x^2+9)(2x+3)(2x-3)$

5. Divide : $15(y+3)(y^2-16)$ by $5(y^2-y-12)$. [NCERT Exemplar]

Sol. Factorising $15(y+3)(y^2-16)$,

We get, $5 \times 3 \times (y+3)(y-4)(y+4)$

On factorising, $5(y^2-y-12)$, we get $5(y^2-4y+3y-12)$

$= 5(y(y-4)+3(y-4)]$

Therefore, on dividing the first expression by the

second expression, we get $\frac{15(y+3)(y^2-16)}{5(y^2-y-12)}$

$= \frac{5 \times 3 \times (y+3)(y-4)(y+4)}{5 \times (y-4)(y+3)}$

$= 3(y+4)$

6. Verify that : $(11pq+4q)^2 - (11p-4q)^2 = 176pq^2$ [NCERT Exemplar]

Sol. L.H.S. $= (11pq+4q)^2 - (11pq-4q)^2$

$= (11pq+4q+11pq-4q) \times (11pq+4q-11pq+4q)$

[using $a^2 - b^2 = (a-b)(a+b)$, here, $a = 11pq+4q$ and $b = 11pq-4q$]

$= (22pq)(8q)$

$= 176pq^2 =$ R.H.S. Hence Verified.

7. Factorise : $p^4+q^4+p^2q^2$ NCERT Exemplar]

Sol. $p^4+q^4+p^2q^2 = (p^2)^2+(q^2)^2+p^2q^2$

$= (P^2)^2 + (9^2)^2 + p^2q^2 + 2p^2q^2 - 2P^2q^2$

[$\because$ Add and subtract $2p^2q^2$]

$= (p^2)^2 + (q^2)^2 2p^2q^2 + p^2q^2 - 2p^2q^2$

$= (p^2 + q^2)^2 – (pq)^2$

[$\because a^2 + b^2 + 2ab = (a + b)^2$]

$= (p^2 + q^2 + pq)(p^2 + q^2 - pq)$

[$\because a^2 - b^2 = (a + b)\ (a\ b)$]

8. Factorise and divide the following :

(a) $(3x^2 -48) + (x-4)$

(b) $(x^4 - 16) \div x^3 + 2x^2 + 4x + 8$ **[NCERT Exemplar]**

Sol. (a) $(3x^2 - 48) \div (x -4) = \frac{3x^2 - 48}{x - 4}$

$= \frac{3(x^2 - 16)}{x - 4}$

$= \frac{3(x^2 - x^4)}{x - 4}$

$= \frac{3(x-4)(x+4)}{(x - 4)}$

[$\because a^2 – b^2 = (a + b)\ (a – b)$]

(b) $(x^4 - 16) \div x^3 + 2x^2 + 4x + 8 = \frac{x^2 - 16}{x^3 + 2x^2 + 4x + 8}$

$\because$ $x^4 – 16 = (x^2)^2 – (4)^2$

$= (x^2 + 4)\ (x^2 – 4)$

$= (x^2 + 4)\ (x^2 – 2^2)$

$= (x^2 + 4)\ (x + 2)\ (x - 2)$

And, $x^3 + 2x^2 + 4x + 18 = x^2\ (x + 2)(x + 2)\ (x -2)$

$= (x + 2)\ (x^2 + 4)$

$\therefore$ $\frac{x^2 - 16}{x^3 + 2x + 4x + 18} = \frac{(x^2 + 4)\ (x + 2)\ (x – 2)}{(x+2)(x^2+4)}$

$= (x -2)$

II. Long answer type questions.

1. Divide $24\ (x^2yz + xy^2z + xyz^2)$ by $8xyz$ using both the methods.

Sol. **$24(x^2yz + xy^2z + xyz^2)$**

$= 2 \text{ x } 2 \text{ x } 2 \text{ x } 3 \text{ x } [(x \text{ x } xxyxz) + (x \text{ x } yxyxz) + (x \text{ x } yxz \text{ x } z)]$

$= 2 \text{ x } 2 \text{ x } 2 \text{ x } 3 \text{ x } x \text{ x } y \text{ x } z \text{ x } (x + y + z) = 8 \text{ x } 3 \text{ x } xyz \text{ x } (x + y + z)$

[By taking out the common factor]

Therefore, $24\ (x^2yz + xy^2z + xyz^2) \div 8xyz = \frac{8 \times 3 \times xyz \times (x + y + z)}{8 \times xyz} = 3(x + y + z)$

Alternately, $24(x^2yz + xy^2 + xyz^2) \div 8xyz = \frac{24x^2yx}{8xyz} + \frac{24xy^2z}{8xyz} + \frac{24xyz^2}{8xyz}$

$= 3x + 3y + 3z = 3(x + y + z)$

2. Factorise $p^4 + q^4 + p^2q^2$. [NCERT Exemplar]

Sol. $p^4 + q^4 + p^2q^2$

Here, we are using identity $(a + b)^2 = a^2 + b^2 + 2ab$

$p^4 + q^4 + p^2q^2 = (p^2)^2 + (q^2)^2 + p^2q^2$

To form the identity we will add and subtract $2p^2q^2$

$= (p^2)^2 + (q^2)^2 + p^2q^2 + 2p^2q^2 + 2p^2q^2$

$= (p^2)^2 + (q^2)^2 + 2p^2q^2 + p^2q^2 - 2p^2q^2 = (p^2 + q^2)^2 – (pq)^2$

Now, we will use identity $a^2 - G^2 = (a + b) (a - b)$

$p^4 + q^4 + p^2q^2 = (p^2 + q^2 + pq) (p^2 + q^2 –pg)$

3. Factorise:

(i) $9x^2 - (3y + z)^2$ **(ii) $a^2 -16a - 80$** **(iii) $a^2 y^3 – 2aby^2 + b^2y$**

Sol. (i) $9x^2 - (3y + z)^2 = (3x)^2 - (3y + z)^2$

Using identity $a^2 - b^2 = (a + b) (a - b)$, so here $a = 3x$, $b = 3y + z$

$(3x)^2- (3y + z)^2 = (3x + 3y + z) (3x - 3y - z)$

(ii) $a^2 -16a - 80$

We know that $- 80 = -20 \times 4$ and $-16 = -20 + 4$ therefore,

$a^2 -16a - 80 = a^2 - 20a + 4a - 80$

$= a(a - 20) + 4(a -20) = (a + 4)(a - 20)$

(iii) $a^2 y^3 – 2aby^2 + b^2y$

In the above equation, y is the common factor.

$a^2y^3 - 2aby^2 + b^2y = y(a^2y^2- 2aby + b^2)$

Now, $a^2 y^2 - 2aby + b^2 = (ay)^2 + b^2 - 2aby$

Using identity $a^2 + b^2 - 2ab = (a - b)^2$

We get, $a^2y^2 - 2aby^2 + b^2 = (ay – b)^2$

Therefore, $a^2y^3 - 2aby^2 + b^2y = y (ay - b)^2$

$= y(ay - b)(ay - b)$

4. Divide

(i) $44(x^4 - 5x^3 - 24x^2)$ by $11x(x - 8)$ **(ii) $(-qrxy + pryz – rxyz) \div (-xyz)$**

Sol. (i) Factorising $44(x^4 – 5x^3 - 24x^2)$, we get

$44(x^4 – 5x^3 -24x^2) = 2 \times 2 \times 11 \times x^2(x^2 - 5x - 24)$

(taking the common factor x^2 out of the bracket)

$= 2 \times 2 \times 11 \times x^2 (x^2 – 8x + 3x - 24)$

$= 2 \times 2 \times 11 \times x^2 [x(x- 8) + 3(x – 8)]$

$= 2 \times 2 \times 11 \times x^2(x + 3)(x – 8)$

Therefore, $44(x^4 - 5x^3 - 24x^2) + 11x(x - 8)$

$$= \frac{2 \times 2 \times 11 \times x \times x \times (x+3) \times (x-8)}{11 \times x \times (x-8)}$$

$= 2 \times 2 \times x(x + 3) = 4x(x + 3)$

(i) (-qrxy + pryz – rxyz) ÷ (-xyz)

Dividing we get,

$$\frac{-ry(qx-pz+xz)}{-xyz} = \frac{r(qx-pz+xz)}{xz}$$

$$= \frac{\text{rqx}}{\text{zx}} - \frac{rpz}{xz} + \frac{xrz}{xz} = \frac{rq}{z} - \frac{rp}{x} + r$$

5. Factorise and divide the following

($2x^3$ -$12x^2$ + $16x$) ÷ (x - 2) (x - 4)

Sol. ($2x^3$ -$12x^2$ + $16x$) ÷ (x - 2) (x - 4)

Factorising $2x^3$ -$12x^2$ + 16x we get,

= $2x(x^2 - 6x + 8)$

Now, factorisingx^2 - $6x$ + 8, we note 8 = 4 x 2 and 4 + 2 = 6. Therefore,

x^2- $6x$ + 8 = x^2 - $4x$ - $2x$ + 8

= $x(x - 4) - 2(x - 4) = (x - 2)(x - 4)$

Hence, ($2x^3$ - $12x^2$ + $16x$) = $2x(x - 2)(x - 4)$

Now, dividing them,

$$\frac{2x(x-2)(x-4)}{(x-2)(x-4)} 2x$$

I. High Order Thinking Skills [HOTS] Questions.

1. (a) Factorise :$x^8 – y^8$

(b) Verify that : $\frac{a^2-b^2}{a+b}$ + - (a - b)

Sol. (a) $x^8 – y^8$= $(x^4)^2$ $(y^4)^2$

= $(x^4 + y^4)$ $(x^4 - y^4)$

= (x^4 + y=) [$(x^2)^2$ $(y^2)^2$]

= $(x^4 + y^4)$ $(x^2 + y^2)$ (x^2-y^2)

= $(x^4 + y^4)$ $(x^2 + y^2)$ (x + y) (x - y)

(b) L.H.S. $= \frac{a^2-b^2}{(\text{a+b})}$

$= \frac{(\text{a}+\text{b})(\text{a}-\text{b})}{(a+b)}$

= (a - b) Hence proved.

II. High Order Thinking Skills [HOTS] Questions.

1. Factorise: $x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x}$ [NCERT Exemplar]

Sol. $x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x}$

$$= \left[x^2 + \frac{1}{x^2} + 2\right] - 3x - \frac{3}{x} = \left[x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x}\right] - 3x - \frac{3}{x}$$

$$= \left[x + \frac{1}{x}\right]^2 - 3\left[x + \frac{1}{x}\right]$$ [Using $a^2 + b^2 + 2ab = (a + b)^2$]

$= \left[x + \frac{1}{x}\right] \left[\left(x + \frac{1}{x}\right) - 3\right]$ [Taking common $(x + \frac{1}{x})$]

$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 3\right)$

2. The sum of square of n natural numbers is $\frac{2n^3 + 3n^2 + n}{6}$.Factorise this expression.

Sol. **Given expression,**

$= \frac{2n^3 + 3n^2 + n}{6} = \frac{2n^2 + 3n + n}{6}$ [Taking n common]

$= \frac{n[2n^2 + 2n + n+1]}{6}$ [We know 3 = 2 + 1 and 2 x 1 = 2]

$= \frac{n[2n\,(n+1)+(n+1)]}{6} = \frac{n(2n+1\,(n+1)}{6}$

3. Using factorisation, find the positive square root of

$\frac{(a+b)^2-(c+d)^2}{(a+b)^2-(c-d)^2} \times \frac{(a+b+c)^2-d^2}{(a+b-c)^2-d^2}$

Sol. $\frac{(a+b)^2-(c+d)^2}{(a+b)^2-(c-d)^2} \times \frac{(a+b+c)^2-d^2}{(a+b-c)^2-d^2}$

Using identity $a^2 - b^2 = (a + b)\ (a - 6)$

Factorising $(a + b)^2 - (c + d)^2 = [(a + b + c + d)(a + b - c - d)$

Factorising $(a + b + c)^2 - d^2 = [(a + b + c + d)\ (a + b + c - d)]$

Factorising $(a + b)^2 - (c - d)^2\ [(a + b + c - d)\ (a + b- c + d))$

Factorising $(a + b - c)^2 - d^2 = [(a + b - c + d)\ (a + b - c - d)]$

Putting the identities together, we get

$\frac{(a+b+c+d)(a+b-c-d)}{(a+b+c-d)(a+b+d-c)} \times \frac{(a+b+c+d)(a+b+c-d)}{(a+b-c+d)(a+b-d-c)} = \frac{(a+b+c+d)^2}{(a+b-c+d)^2}$

Now the positive square root is

$\frac{\sqrt{(a+b+c+d)^2}}{\sqrt{(a+b-c+d)^2}} = \frac{a+b+c+d}{a+b-c+d}$

4. Factorise $6\sqrt{5x^2} - 2x - 4\sqrt{5}$

Sol. $6\sqrt{5x^2} - 2x - 4\sqrt{5}$

We know, $6\sqrt{5x^2} - 2x - 4\sqrt{5}$ = -120 and -12 + 10 = -2. Therefore,

$= 6\sqrt{5x^2} - 12x + 10x - \sqrt{5}$ (Splitting the middle term)

$= 6x(\sqrt{5x^2} - 2) + 2\sqrt{5}(\sqrt{5x} - 2) - (6x + 2\sqrt{5})(\sqrt{5x} - 2)$

I. Value based question.

1. **(a) Factorise :$x^2 + 15x + 56$**

(b) If x = 3 and y = 2, then verify that :$x^2 - y^2 = (x + y)\ (x - y)$

Sol. (a) $x^2 + 15x + 56 = x^2 + (8 + 7)x + 56$

$= x^2+8x+7x+56$

$= x\,(x + 8) + 7\,(x + 8)$

$= (x+8)(x+7)$

(b) Putting, x = 3 and y = 2, then

L.H.S. = $x^2 - y^2$

$= 3^2 - 2^2$

$= 9 - 4 = 5$

and R.H.S. = $(x + y)(x - y)$

$= (3 + 2)(3 - 2)$

$= 5 \times 1 = 5$

L.HS. = R.H.S. Hence Proved.

2. (a) Factorise : $15x^2 - 26x + 8$

(b) If a = 2 and b = 1, then verify that : $(a - b)^2 = a^2 + b^2 - 2ab$.

Sol. (a) $15x^2 - 26x + 8 = 15x^2 - (20 + 6)x + 8$

$= 15x^2 - 20x - 6x + 8$

$= 5x(3x - 4) - 2(3x - 4)$

$= (3x - 4)(5x - 2)$

(b) Putting, a = 2 and b = 1, then

L.H.S. $= (a - b)^2 = (2 - 1)^2$

$= (2 - 1)^2 = 1^2 = 1$

R.H.S. $= a^2 + b^2 - 2ab$

$= 2^2 + 1^2 - 2 \times 2 \times 1$

$= 4 + 1 - 4 = 1$

L.H.S. = R.H.S. Hence Proved.