



Grade X

Lesson : 1 REAL NUMBER

**Objective Type Questions**

**I. Multiple choice questions**

1. If two positive integers  $a$  and  $b$  are written as  $a = x^3y^2$  and  $b = xy^3$ , where  $x, y$  are prime numbers then HCF ( $a, b$ ) is
  - a)  $xy$
  - b)  $xy^3$
  - c)  $x^3y^3$
  - d)  $x^3y^2$
2. If two positive integers  $p$  and  $q$  can be expressed as  $p = ab^2$  and  $q = ab$  and  $q = a^3b$ ; where  $a, b$  being prime numbers, then LCM ( $p, q$ ) is equal to
  - a)  $ab$
  - b)  $a^2b^2$
  - c)  $a^3b^2$
  - d)  $a^2b^3$
3. The HCF and the LCM of 12, 21, 15 respectively are
  - a) 3, 140
  - b) 12, 420
  - c) 3, 420
  - d) 420, 3
4. Two alarm clocks ring their alarms at regular intervals of 50 seconds and 48 seconds. If they first beep together at 12 noon at what time will they first beep again for the time?
  - a) 12.20 pm
  - b) 12.12 pm
  - c) 12.11 pm
  - d) none of these
5. If  $A = 2n + 13$ ,  $B = n + 7$  where  $n$  is a natural number, then HCF of  $A$  and  $B$  is
  - a) 2
  - b) 1
  - c) 3
  - d) 4
6. There are 576 boys and 448 girls in a school that are to be divided into equal sections of either boys or girls alone. The total number of sections thus formed are:
  - a) 22
  - b) 16
  - c) 36
  - d) 21
7. The HCF of 2472, 1284 and a third number  $N$  is 12. If their LCM is  $2^3 \times 3^2 \times 5 \times 103 \times 107$ , then the number  $N$  is.
  - a)  $22 \times 32 \times 7$
  - b)  $22 \times 33 \times 103$
  - c)  $22 \times 32 \times 5$
  - d)  $24 \times 32 \times 11$
8. Two natural numbers whose difference is 66 and the least common multiple is 360, are:
  - a) 120 and 54
  - b) 90 and 24
  - c) 180 and 114
  - d) 130 and 64





14. Which of the following is terminating ?

- a)  $\frac{7}{14}$                       b)  $\frac{3}{150}$                       c)  $\frac{5}{50}$                       d) All of these

15. Which of the following cannot be the unit place digit in expansion of  $(5678.3)^n$

$n \in \mathbb{N}$ ?

- a) 3                      b) 9                      c) 7                      d) 4

16. H.C.F. of 135 and 225 is

- a) 3                      b) 15                      c) 30                      d) 45

17. Find L.C.M. of 135 and 225 is

- a) 765                      b) 225                      c) 135                      d) None of the above

18. Which of given result is not rational?

- a)  $(3 + \sqrt{5})(3 - \sqrt{5})$     b)  $\sqrt{3} \times \sqrt{12}$                       c)  $\frac{\sqrt{5}}{\sqrt{625}}$                       d) None of the above

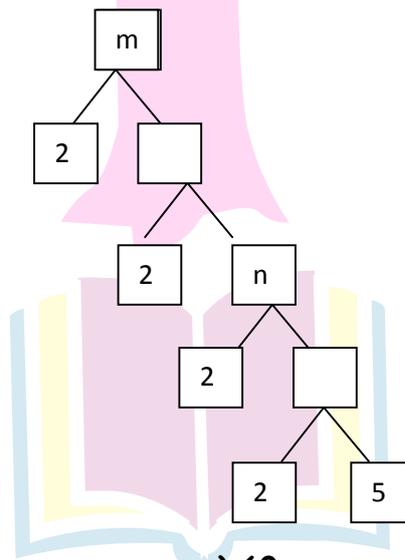
19. Number of prime factors of 5005 is

- a) 6                      b) 4                      c) 3                      d) 2

20. Which of the following is not even ( $q \in \mathbb{N}$ )?

- a)  $2q + 1$                       b)  $4q + 2$                       c)  $6q + 4$                       d) All of these

21. From given factor free find value of  $(m - n)$



- a) 160                      b) 40                      c) 60                      d) 320

22. Value of  $(5 - \sqrt{2})(5 + \sqrt{2})$  is

- a) natural number    b) whole number    c) rational number    d) All of these



23. Decimal representation of  $\frac{2}{3}$  is
- a)  $0.\bar{3}$                       b)  $0.\bar{4}$                       c)  $0.\bar{6}$                       d)  $0.\bar{7}$
24. Which of the following number cannot be as part of prime factorisation of denominator of rational number (Which is non-terminating)?
- a) 2                      b) 5                      c) 3                      d) None of these
25. Units place digit in the expansion of  $(576895)^{201}$  is
- a) 0                      b) 1                      c) 3                      d) 5
26. L.C.M. of 336 and 54 is
- a) 324                      b) 3024                      c) 3124                      d) 3224
27. Decimal expansion of  $\frac{1}{3}$  is
- a) 0.3                      b) 0.33                      c) 0.333                      d)  $0.\bar{3}$
28. Number 12673 is divisible by
- a) 19                      b) 23                      c) 29                      d) All of these
29. If x and y both are odd numbers, then  $(x^2 + y^2)$  is divisible by
- a) 2                      b) 4                      c) 12                      d) All are possible
30. For two positive integers a and b ( $a > b$ ) if  $a = b \times q + r$ , then value of r may be
- a) zero                      b) always less than b                      c)  $0 \leq r < b$                       d) None of these
31. Euclid's Division Lemma is used to calculate
- a) H.C.F.                      b) L.C.M.                      c) Both (a) and (b)                      d) None of these
32. L.C.M. of smallest prime and smallest composite number is
- a) 1                      b) 2                      c) 3                      d) 4
33. Length, breadth and height of a cuboidal room are 8.50 m 6.25 m and 4.75m respectively. The length of largest rod which can be used to measure all the three dimensions is.
- a) 5 cm                      b) 10cm                      c) 15 cm                      d) 25 cm
34. If  $n \in \mathbb{N}$  and  $(n^2 - 1)$  is always divisible by 8 for n to be
- a) even number                      b) odd number                      c) zero                      d) None of these



35. Number 429 is divisible by

- a) 3                      b) 11                      c) 13                      d) All of these

36. Two containers has capacity of 850 l and 680 l. Capacity of biggest container which can measure the liquid of both containers?

- a) 17 l                      b) 85 l                      c) 170 l                      d) None of these

37. Number of decimal places for division of  $\frac{15}{1600}$  is

- a) 4                      b) 5                      c) 6                      d) 8

38. Convert  $3.2\bar{3}$  into form of  $\frac{p}{q}$

- a)  $\frac{97}{30}$                       b)  $\frac{291}{90}$                       c)  $\frac{582}{180}$                       d) All of these

39. Which of the following is non-terminating decimal?

- a)  $\sqrt{2}$                       b)  $\frac{1}{\sqrt{2}}$                       c)  $\sqrt{\frac{1}{9}}$                       d) All of these

40. Which of the given is non-terminating repeating decimal?

- a)  $\sqrt{\frac{1}{4}}$                       b)  $\sqrt{\frac{1}{25}}$                       c)  $\sqrt{\frac{1}{225}}$                       d)  $\sqrt{\frac{1}{100}}$

41.  $\pi$  is

- a) irrational                      b) rational                      c)  $\frac{22}{7}$                       d) None of these

42. Product of a rational number with an irrational number is always

- a) rational number  
b) irrational number  
c) may be any rational or irrational number  
d) None of these

43. If  $n \in \mathbb{N}$ , then  $11^n - 6^n$  is always divisible by

- a) 11                      b) 6                      c) 15                      d) 17

44. Find L.C.M. of a and b if  $a = x^3y^2z$  and  $b = x^2y^3z^4$

- a)  $xyz$                       b)  $x^2y^2z$                       c)  $x^3y^2z^4$                       d) None of these

45. Largest positive number, which can divide 398, 436, 542 leaving remainder 7, 11 and 15 respectively.

- a) 10                      b) 12                      c) 15                      d) 17

46. L.C.M. of 6, 72 and 120 is

- a) 120                      b) 240                      c) 360                      d) 720

47.  $(4)^n$   $n \in \mathbb{N}$  cannot end with digit

- a) 0                      b) 1                      c) 5                      d) 7

48.  $\frac{p}{q}$  form of  $43.\overline{123456789}$  is

- a) 1                      b) 0                      c) 43                      d)  $\frac{943123456789}{999999999}$

**III. Multiple choice questions**

1. For some integer  $m$ , every even integer is of the form

- a)  $m$                       b)  $m + 1$                       c)  $2m$                       d)  $2m + 1$

2. If  $n$  is an even natural number then the largest natural number by which  $n(n + 1)(n + 2)$  is divisible, is

- a) 6                      b) 8                      c) 12                      d) 24

3. The product of two consecutive positive integers is divisible by  $2!$ .

- a) True                      b) false  
c) Can't say                      d) Partially True / False

4. The number  $4^n$ , where  $n$  is a natural number, ends with the digit 0 for any natural number  $n$ .

- a) True                      b) False  
c) Can't say                      d) Partially

5.  $12^n$  ends with the digit 0 or 5 for natural number  $n$ .

- a) 2                      b) 3                      c) No value                      d) 4

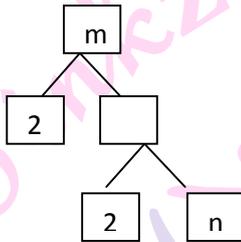
6.  $(3 \times 5 \times 7) + 7$  is a

- a) Prime number                      b) Composite number  
c) Can't say                      d) None of these

7. The number  $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 5$  is \_\_\_\_\_ number.  
 a) Prime                      **b) Composite**                      c) Can't say                      d. None of these

8. The total number of factors of a prime number is is.  
 a) 1                              **b) 2**                              c) 0                              d. 3

9. The values of x and y is the given figure are



**a) 7, 13**                      b) 13, 7                      c) 9, 12                      d) 12,9

10. HCF of 96 and 404 is equal to \_\_\_\_\_  
 a) 2                              b) 3                              **c) 4**                              d) 5

11. Total number of distinct primes in the prime factorization of number 27300.  
**a) 5**                              b) 7                              c) 13                              d) 21

12. The unit place digit of HCF of  $2^3 \times 3^2 \times 5^3 \times 7$ ,  $2^3 \times 3^3 \times 5^2 \times 7^2$  and  $3 \times 5 \times 7 \times 11$  is  
 a) 70                              **b) 105**                              c) 175                              d) 225

13. If two positive integers a and b are written as  $a = x^3 y^2$  and  $b = x y^3$ , where x, y are prime numbers, then HCF (a, b) is  
 a) xy                              **b)  $x y^2$**                               c)  $x^3 y^3$                               d)  $x^2 y^2$

14. Write the HCF of the smallest composite number and the smallest prime number  
 a) 1                              **b) 2**                              c) 3                              d) 4

15. Two numbers are in the ratio of 15: 11 . If their HCF is 13, then numbers will be  
**a) 195 and 143**                              b) 190 and 140  
 c) 185 and 163                              d) 185 and 143

16. Product of two coprime numbers is 117 their LCM should be \_\_\_\_\_.  
 a) 1                              **b) 117**



28. The LCM and the HCF of two numbers are 1001 and 7 respectively. How many such pairs are possible?

- a) 0                      b) 1                      c) 2                      d) 7

29. If  $p$  is prime, then HCF and LCM of  $p$  and  $p + 1$  would be

- a) HCF =  $p$ , LCM =  $p + 1$                       b) HCF =  $p(p + 1)$  LCM = 1  
c) HCF = 1, LCM =  $p(p + 1)$                       d) None of the above

30. The HCF and LCM of 12, 21 and 15 respectively, are

- a) 3, 140                      b) 12, 420                      c) 3, 420                      d) 420, 3

31. In which of the following is a irrational number?

- a)  $\frac{22}{7}$                       b) 3.1416                      c)  $3.\overline{1416}$                       d) 3. 141441444...

32. The number of irrational numbers between 15 and 18 is infinite.

- a) True                      b) False                      c) Can't say                      d) Partially True / False

33. The product of a non-zero rational and an irrational number is

- a) always irrational                      b) always rational  
b) rational of irrational                      d) one

34.  $3.\overline{27}$  is

- a) an integer number                      b) a rational number  
c) an irrational number                      d) None of these

35. A rational number  $p/q$  has a terminating decimal expansion if prime factorization of  $q$  have

- a) 3                      b) 2                      c) 5                      d) Both (b) and (c)

36. If  $\frac{13}{125}$  is a rational number, then decimal expansion of it, which terminates

- a) 0. 104                      b) 1. 01                      c) 0. 0104                      d) 0. 140

37. On the basis of form  $2^m \times 5^n$  of denominator that  $\frac{1458}{1250}$  will be expanded in decimal up to places will be

- a) one                      b) two                      c) three                      d) four



- P O R S  
 a) 2 4 1 3  
 c) 1 2 3 4

- P O R S  
 b) 2 1 4 3  
 d) 1 2 3 4

**Fill in the blanks**

1. If a number can't be expressed in the form  $\frac{p}{q}$  where  $p$  and  $q$  are integers and  $q \neq 0$ , then it is known as \_\_\_\_\_ number  
**Irrational**
2. If a rational number  $\frac{p}{q}$ , where  $q \neq 0$  and  $p, q$  are coprimes, has a terminating decimal expansion then  $q$  has prime factors \_\_\_\_\_ and \_\_\_\_\_.  
**2 and 5**
3. If a rational number  $\frac{p}{q}$  (in lowest form) has a non-terminating and repeating decimal expansion, then prime factorisation of  $q$  is of the form \_\_\_\_\_.  
 **$2^m \times 5^n$  where  $m, n$  are non-negative integers**
4. Euclid's division algorithm is a technique to Compute \_\_\_\_\_ the integers.  
**H.C.F.**
5. H.C.F. of two or more prime numbers is always \_\_\_\_\_.  
**1**
6.  $7 \times 11 \times 13 + 3$  is a \_\_\_\_\_ number. (Prime / composite/irrational)  
**Composite**
7. If product of two coprimes is 765, then their number L.C.M. \_\_\_\_\_.  
**765**
8. Sum of a rational and irrational number is always \_\_\_\_\_ number.  
**irrational**
9. Product of any irrational number with a rational number ' $x$ ' is always rational.  
 Then,  $x$  is \_\_\_\_\_.  
**0**

10. Any number ending with '0' must have \_\_\_\_\_ and \_\_\_\_\_ as its prime factors.

2, 5

### I Very Short Answer Type Questions

1. If two positive integers a and b are written  $a = x^3y^2$  and  $b = xy^3$ , where x, y are prime numbers, then HCF (a, b) is

- a) xy                      b)  $xy^2$                       c)  $x^3y^3$                       d)  $x^2y^2$

Also find LCM of (a, b)

$$a = x^3y^2 \text{ and } b = xy^3$$

$$\Rightarrow a = x \times x \times x \times y \times y$$

$$\text{and } b = xy \times y \times y$$

$$\therefore \text{HCF (a, b)} = x \times y \times y = x \times y^2 = xy^2$$

$$\text{LCM} = x^3y^3$$

2. If two positive integers p and q can be expressed as  $p = ab^2$  and  $q = a^2b$ ; where a, b being prime numbers then LCM ( p, q) is equal to

- a. ab                      b.  $a^2b^2$                       c.  $a^3b^2$                       d)  $a^2b^3$

3. The HCF and LCM of two numbers are 33 and 264 respectively. When the first number is completely divided by 2 the quotient is 33. The other number is \_\_\_\_\_.

$$\text{First number} = 2 \times 33 = 66$$

$$\therefore \text{other number} = \frac{\text{HCF} \times \text{LCM}}{\text{1st number}} = \frac{33 \times 264}{66} = 132$$

4. Find the LCM of smallest prime and smallest odd composite natural number

$$\text{Smallest prime number} = 2$$

$$\text{Smallest composite odd number} = 9$$

$$\text{LCM of 2 and 9} = 2 \times 9 = 18$$

5. Decompose 32760 into prime factors

$$32760 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 13$$

$$= 2^3 \times 3^2 \times 5 \times 7 \times 13$$

6. Write the sum of exponents of prime factors in the prime factorisation of 250.

$$250 = 2 \times 5^3$$

$$\therefore \text{Sum of exponents} = 1 + 3 = 4$$

7. What is the HCF of smallest prime number and the smallest composite number?

$$\text{The smallest prime number} = 2$$

$$\text{The smallest composite number} = 4$$

$$\therefore \text{HCF of 2 and 4} = 2$$

8. If the prime factorisation of a natural number N is  $2^4 \times 3^4 \times 5^3 \times 7$ , write the number of consecutive zeroes in N.

$$\text{Number of consecutive zeroes} = \text{zeroes in } 2^3 \times 5^3 = \text{Zeroes in } (10)^3 = 3$$

9. If product of two numbers is 3691 and their LCM is 3691, find their HCF.

$$\text{HCF} = \frac{\text{Product of two numbers}}{\text{LCM}} = \frac{3681}{3691} = 1$$

10. After how many places of decimal, the decimal expansion of  $\frac{43}{2^4 \times 5^3}$  will terminate?

$$\text{Given } \frac{43}{2^4 \times 5^3} \text{ is in the lowest form and power of } 2 = 4, \text{ Power of } 5 = 3, \text{ Max, } [4, 3] = 4$$

$$\therefore \frac{43}{2^4 \times 5^3} \text{ will terminate after 4 places of decimal.}$$

11. What is the exponent of 3 in the prime factorisation of 864.

Making prime factors of 864.

$$\Rightarrow 864 = 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 3^3 \times 2^5$$

$$\therefore \text{Exponent of 3 in prime factorisation of 864} = 3.$$

12. Express 10010 and 140 as prime factors.

$$\text{Prime factors of 10010} = 2 \times 5 \times 7 \times 11 \times 13$$

$$\text{Prime factors of 140} = 2 \times 2 \times 5 \times 7$$

13. What is the H.C.F of smallest prime and the smallest composite number ?

$$\text{Smallest prime number} = 2 = 2^1$$

Smallest composite number =  $4 = 2^2$

H.C.F. (2, 4) =  $2^1 = 2$

14. Write a rational number between  $\sqrt{5}$  and  $\sqrt{6}$

$$\sqrt{5} = 2.236\dots \quad \sqrt{6} = 2.449$$

∴ A rational no between 2.24 and 2.44

tc. Sol  $\sqrt{5} = 2.236\dots$  and  $\sqrt{6} = 2.449$  (approximately) is 2.3 or 2.31 or 2.32 etc

Note: Take the lower limit slightly greater than  $\sqrt{5}$  and upper limit slightly lesser than  $\sqrt{6}$

⇒ One number between  $\sqrt{5}$  and  $\sqrt{6} = 2.3$

15. If  $p, q$  are two prime numbers then what is the HCF and LCM of  $p$  and  $q$ ?

$$\text{HCF}(p, q) = 1$$

$$\text{and LCM}(p, q) = pq$$

16. A rational number in its decimal expansion is 623.6051. What can you say about the prime factors of  $q$ , when this number is expressed in the form  $\frac{p}{q}$ ? Give reasons.

Since 623.6051 is a terminating decimal number, so  $q$  must be of the form  $2^m 5^n$  where  $m, n$  are natural numbers.

17. 'Product of two irrational numbers is always an irrational number'. Negate the statement by giving counter example.

Take  $2 - \sqrt{5}$  and  $2 + \sqrt{5}$  both are irrational. Their product

$$(2 - \sqrt{5})(2 + \sqrt{5})$$

$$2^2 - (\sqrt{5})^2 = 4 - 5 = -1$$

Which is a rational number.

18. Can two numbers have 24 as their HCF and 7290 as their LCM? Give reasons.

No, because HCF always divides LCM but here 24 does not divide 7290.

Note: If  $b$  is a factor of  $a$  then  $\text{HCF}(a, b) = b$  for instance in Question 8, Pg-14; 63 is a factor of 693.

## I Short Answer Type Questions

1. Two numbers are in the ratio 21: 17. If their HCF is 5, find the numbers

Let numbers are  $21x$  and  $17x$

Now common factor of  $21x$  and  $17x = x$

Also HCF = 5

$$\Rightarrow x = 5$$

$\therefore$  numbers are  $21 \times 5$  and  $17 \times 5$  i.e. 105 and 85

2. The HCF of two numbers is 29 and other two factors of their LCM are 16 and 19. Find the larger of the two numbers.

HCF of the two numbers is 29

$\therefore$  Numbers are  $29 \times a$  and  $29 \times b$  where  $a$  and  $b$  are co-prime.

Now other two factors of the LCM are 16 and 19.

$$\Rightarrow 29 \times 16 \times 19 = 29 \times a \times b$$

$$\Rightarrow a = 16 \text{ and } b = 19$$

So, larger of the two number is  $29 \times 19 = 551$

3. Three numbers are in the ratio 2:5:7. Their LCM is 490. Find the square root of the largest number.

Let numbers are  $2x$ ,  $5x$  and  $7x$

$\therefore$  LCM of  $2x$ ,  $5x$  and  $7x = 2 \times 5 \times 7 \times x$

Also LCM = 490

$$\Rightarrow 2 \times 5 \times 7 \times x = 490$$

$$\Rightarrow x = 7$$

So numbers are  $2 \times 7$ ,  $5 \times 7$  and  $7 \times 7 = 14$ , 35 and 49

Largest number = 49

$\therefore$  The square root of largest number =  $\sqrt{49} = 7$

4. If least prime factor of a is 5 and least prime factor of b is 13, then what is the least prime factor of a + b?

Least prime factor of a = 5

∴ a is odd

Also least prime factor of b = 13

∴ b is also odd

Now a + b = sum of two odd numbers = even number

∴ Least prime factor of a + b is 2

5. Can we have any  $n \in \mathbb{N}$ , where  $7^n$  ends with the digit zero?

For units digit to be 0,  $7^n$  should have 2 and 5 as its prime factors, but  $7^n$  does not contain 2 and 5 as its prime factors. Hence  $7^n$  will not end with digit 0 for  $n \in \mathbb{N}$

6. Given that  $\sqrt{2}$  is irrational, prove that  $(5 + 3\sqrt{2}) = \frac{p}{q}$  be a rational number where p and q have no common factor other than  $q \neq 0$ .

$$\Rightarrow 3\sqrt{2} = \frac{p}{q} - 5 \Rightarrow \sqrt{2} = \frac{p-5q}{3q}$$

For any values of p and q ( $q \neq 0$ ), RHS  $\frac{p-5q}{3q}$  is rational,

This contradicts the fact, So our assumption is wrong

∴  $5 + 3\sqrt{2}$  is an irrational number.

7. 3 bells ring at an interval of 4, 7 and 14 minutes. All three bells rang at 6 am, when the three bells will be ring together next?

We know that, the three bells again ring together on that time which is the LCM of individual time of each bell

$$4 = 2 \times 2$$

$$7 = 7 \times 1$$

$$14 = 2 \times 7$$

$$\text{LCM} = 2 \times 2 \times 7 = 28$$

The three bells will ring together again at 6: 28



8. Find the HCF of 1260 and 7344 using Euclid's algorithm. Or Using Euclid's Algorithm

$$7344 = 1260 \times 5 + 1044$$

$$1260 = 1044 \times 1 + 216$$

$$1044 = 216 \times 4 + 180$$

$$216 = 180 \times 1 + 36$$

$$180 = 36 \times 5 + 0$$

$\therefore$  HCF of 1260 and 7344 is 36.

9. Show that every positive odd integer is of the form  $(4q + 1)$  or  $(4q + 3)$ , where  $q$  is some integer.

Using Euclid's Algorithm

$$a = 4q + r, 0 \leq r < 4$$

$$\Rightarrow a = 4q, a = 4q + 1, a = 4q + 2 \text{ and } a = 4q + 3.$$

Now  $a = 4q$  and  $a = 4q + 2$  are even numbers.

Therefore when  $a$  is odd, it is of the form

$$a = 4q + 1 \text{ or } a = 4q + 3 \text{ for some integer } q.$$

10. Write the smallest number which is divisible by both 306 and 657.

Smallest number divisible by 306 and 657

$$= \text{LCM}(306, 657)$$

$$\text{LCM}(306, 657) = 22338$$

The smallest number that is divisible by two numbers is obtained by finding the LCM of these numbers Using Euclid's Algorithm

$$657 = 306 \times 2 + 45$$

$$306 = 45 \times 6 + 36$$

$$45 = 36 \times 1 + 9$$

$$36 = 9 \times 4 + 0$$

$$\therefore \text{HCF}(657, 306) = 9$$

$$\text{LCM} = \frac{\text{Product of two numbers}}{\text{HCF}(657, 306)}$$



Next Generation School

$$\frac{657 \times 3}{9} = 657 \times 34$$

$$\text{LCM}(657, 306) = 22338$$

Hence, the smallest number which is divisible by 306 and 657 is 22338

11. The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, find the other number.

Since,  $\text{HCF} \times \text{LCM} = \text{Product of two numbers}$

$$\text{Then, } 9 \times 360 = 45 \times 2^{\text{nd}} \text{ number}$$

$$2^{\text{nd}} \text{ number} = \frac{(9 \times 360)}{45}$$

$$\text{Thus, } 2^{\text{nd}} \text{ number} = 72$$

12. If two positive integers  $p$  and  $q$  are written as  $p = a^2b$  and  $q = a^3b^3$ , where  $a$  and  $b$  are prime numbers then verify.

$$\text{LCM}(p, q) \times \text{HCF}(p, q) = p \cdot q.$$

$$\text{Since } \text{LCM}(p, q) = a^3b^3$$

$$\text{and } \text{HCF}(p, q) = a^2b$$

$$\text{Hence } \text{LCM}(p, q) \times \text{HCF}(p, q) = a^3b^3 \times a^2b$$

$$= pq$$

Hence Verified

13. Explain whether  $3 \times 12 \times 101 + 4$  is a prime number or a composite number.

$$3 \times 12 \times 101 + 4 = 4(3 \times 3 \times 101 + 1)$$

$$= 4(909 + 1)$$

$$= 4(910)$$

$$= 2 \times 2 \times 2 \times 5 \times 7 \times 13$$

$$= \text{a composite number}$$

[Product of more than two prime factors]

14. Find the HCF and LCM of 90 and 144 by the method of prime factorization.

$$\text{Since } 90 = 2 \times 3^2 \times 5$$

$$\text{and } 144 = 2^4 \times 3^2$$

$$\text{Hence } \text{HCF} = 2 \times 3^2 = 18$$

$$\text{and } \text{LCM} = 2^4 \times 3^2 \times 5 = 720$$

15. Find the HCF of 1260 and 7344 using Euclid's algorithm.

Or

Show that every positive odd integer is of the form  $(4q + 1)$  or  $(4q + 3)$ , where  $q$  is some Integer

Taking  $a = 7344$  and  $b = 1260$

Applying Euclid's Division Algorithm

$$7344 = 1260 \times 5 + 1044$$

$$1260 = 1044 \times 1 + 216$$

$$1044 = 216 \times 4 + 180$$

$$216 = 180 \times 1 + 36$$

$$180 = 36 \times 5 + \boxed{0} \leftarrow \text{Stop!}$$

HCF of 1260 and 7344 is 36.

Or

Apply Euclid's Division Lemma to  $a$  and  $b = 4$ .

$$a = 4q + r, 0 \leq r < 4$$

$$\Rightarrow a = 4q, a = 4q + 1, a = 4q + 2 \text{ and } a = 4q + 3.$$

Now,  $a = 4q$ ,  $a = 4q + 2$  and  $a = 4q + 3$  are even numbers as both are divisible by 2.

But,  $4q + 1$  and  $4q + 3$  are not divisible by 2.

Therefore, when  $a$  is odd, it is of the form  $a = 4q + 1$  or  $a = 4q + 3$  for some integer  $q$ .

16. Show that  $\frac{3 + \sqrt{7}}{5}$  is an irrational number, given 5 that  $\sqrt{7}$  is irrational.

Or Prove that  $n^2 + n$  is divisible by 2 for any positive integer  $n$ .

Let, if possible  $\frac{3 + \sqrt{7}}{5}$  is rational.

Thus,  $\frac{3 + \sqrt{7}}{5} = \frac{p}{q}$  where  $p$  and  $q$  are integers and  $q \neq 0$

$$\Rightarrow 3q + \sqrt{7}q = 5p$$

$$\Rightarrow \sqrt{7}q = 5p - 3q$$

$$\Rightarrow \sqrt{7} = \frac{5p-3q}{q}$$

Since, difference of integers is also an integer.

$\therefore 5p - 3q$  is an integer.

$$\Rightarrow \frac{5p-3q}{q} \text{ is rational number.}$$

But LHS is  $\sqrt{7}$ , which is irrational.

Thus, irrational number = Rational number which is a contradiction.

Thus, our supposition is wrong.

Hence,  $\frac{3+\sqrt{7}}{5}$  is an irrational number.

Or

$$n^2 + n = n(n+1)$$

There arise two cases.

Case I: When  $n$  is even.

Then  $(n+1)$  is odd.

Since, even  $\times$  odd = even

$\therefore$  Product  $n(n+1)$  is even i.e. divisible by 2.

Case II: When  $n$  is odd.

Then  $(n+1)$  is even.

Since, odd  $\times$  even = even.

$\therefore n(n+1)$  is even i.e. divisible by 2.

So, for any positive integer,  $(n^2 + n)$  is always divisible by 2.

**17. A positive integer  $n$  when divided by 9, gives 7 as remainder. Find the remainder when  $(3n-1)$  is divided by 9.**

Here  $n$  can be written as  $9k + 7$ , where  $k \in \mathbb{N}$

$$\text{Now } 3n-1 = 3(9k+7) - 1 = 27k + 20$$

Applying Euclid's division lemma on  $(27k + 20)$  and 9, we have

$$27k+20 = 9 \times (3k+2) + 2;$$

where  $k \in \mathbb{N}$

Thus, 2 is the remainder.

**18. Show that any positive even integer can be written in the form  $6q$ ,  $6q + 2$  or  $6q + 4$ , where  $q$  is an integer?**

Let take 'a' as any positive even integer and  $b = 6$  Then using Euclid algorithm we get

$$a = 6q + r$$

Here  $r$  is remainder and value of  $q$  is more than or equal to 0 and  $r = 0, 1, 2, 3, 4, 5$  because  $0 < r < b$  and the value of  $b$  is 6.

So total possible forms will be  $6q + 0$ ,  $6q + 2$ ,  $6q + 4$   $6q + 0$

$6$  is divisible by 2 so, it is an even number.  $6q + 2$

$6$  is divisible by 2 and 4 is also divisible by 2

So, it is an even number.

Hence, any positive even integer can be written in the form  $6q$ ,  $6q + 2$  or  $6q + 4$  and so on.

**19. In a school, the duration of a period in junior section is 40 minutes and in senior section is 1 hour. If the first bell for each section rings at 9:00 a.m., when will the two bells ring together again?**

$$1 \text{ hour} = 60 \text{ minutes}$$

$$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$$

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$$

$$\therefore \text{LCM}(40, 60) = 2^3 \times 3 \times 5 = 120$$

$$120 \text{ minutes} = 2 \text{ hours}$$

Hence, the two bells will ring together again at  $9:00 + 2:00 = 11:00$  a.m.

**20. Explain why  $(17 \times 11 \times 2 + 17 \times 11 \times 5)$  is a composite number?**

$$17 \times 11 \times 2 + 17 \times 11 \times 5 = 17 \times 11 \times 5$$

$$= 17 \times 11 \times 7$$

Since,  $17 \times 11 \times 2 + 17 \times 11 \times 5$  can be expressed as a product of primes, therefore, it is a composite number.

## II Short Answer Type Questions

1. Using Euclid's Division Algorithm find the HCF of 726 and 275.

Euclid's division lemma

$$726 = 275 \times 2 + 176$$

$$275 = 176 \times 1 + 99$$

$$176 = 99 \times 1 + 77$$

$$99 = 77 \times 1 + 22$$

$$77 = 22 \times 3 + 11$$

$$22 = 11 \times 2 + 0$$

Thus, HCF = 11

2. Show that exactly one of the number  $n$ ,  $n + 2$  or  $n+4$  is divisible by 3

Let  $n$  be any positive integer and  $b = 3$

Then,  $n = 3q + r$

where,  $q$  is the quotient and  $r$  is the remainder and

$$0 \leq r < 3$$

So, the remainders may be 0, 1 or 2 and  $n$  may be in

the form of  $3q, 3q+1, 3q+2$

Let  $n = 3q, 3q+1$  or  $3q+2$ .

(i) When  $n = 3q$

$\Rightarrow n$  is divisible by 3.

$$n+2 = 3q+2$$

$\Rightarrow n+2$  is not divisible by 3.

$$n+4 = 3q+4 = 3(q+1) + 1$$

$\Rightarrow n+4$  is not divisible by 3. **1**

(ii) When  $n = 3q+1$

$\Rightarrow n$  is not divisible by 3.

$$n+2 = (3q+1) + 2 = 3q+3 = 3(q+1)$$

$\Rightarrow n+2$  is divisible by 3.

$$n+4 = (3q+1) + 4 = 3q+5 = 3(q+1) + 2$$

$\Rightarrow n+4$  is not divisible by 3. **1**

(iii) When  $n = 3q+2$

$\Rightarrow n$  is not divisible by 3.

$$n + 2 = (3q + 2) + 2 = 3q + 4 = 3(q + 1) + 1$$

$\Rightarrow n + 2$  is not divisible by 3.

$$n + 4 = (3q + 2) + 4 = 3q + 6 = 3(q + 2)$$

$\Rightarrow n + 4$  is divisible by 3.

Hence, exactly one of the numbers  $n$ ,  $n + 2$  or  $n + 4$  is divisible by 3. 1

**3. Find the HCF of 180, 252 and 324 by Euclid's Division algorithm.**

Since,  $324 = 252 \times 1 + 72$

$$252 = 72 \times 3 + 36$$

$$72 = 36 \times 2 + 0$$

$$\text{HCF}(324, 252) = 36$$

$$180 = 36 \times 5 + 0$$

$$\text{HCF}(36, 180) = 36$$

HCF of 180, 252 and 324 is 36.

**4. Use Euclid's division algorithm to find HCF of 4052 and 12576.**

$\therefore 12576 > 4052$ , apply the Euclid's division

lemma to 12576 and 4052, to get

$$12576 = 4052 \times 3 + 420$$

Since  $420 \neq 0$ , so apply E.D.L. to 4052 and 420

to get  $4052 = 420 \times 9 + 272$

Since,  $272 \neq 0$ , so apply E.D.L. to 420 and 272

$$420 = 272 \times 1 + 148$$

Since,  $148 \neq 0$ , so apply E.D.L. to 272 and 148

to get  $272 = 148 \times 1 + 124$

Since,  $124 \neq 0$ , so apply E.D.L. to 148 and 124

$$148 = 124 \times 1 + 24$$

Since,  $24 \neq 0$ , so apply E.D.L. to 124 and 24

$$124 = 24 \times 5 + 4$$

Since,  $4 \neq 0$ , so apply E.D.L. to 24 and 4

$$24 = 4 \times 6 + 0 \quad \leftarrow \text{Stop!}$$

Since, remainder has become zero.

$$\therefore \text{HCF}(12576, 4052) = 4.$$

5. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.

According to the statement of the question, we have

$$\text{LCM of two numbers} = 14 \times \text{HCF of two numbers}$$

$$\text{Also } \text{LCM} + \text{HCF} = 600$$

$$\Rightarrow 14 \times \text{HCF} + \text{HCF} = 600$$

$$\Rightarrow 15 \text{ HCF} = 600$$

$$\Rightarrow \text{HCF} = 40$$

$$\therefore \text{LCM } 14 \times 40 = 560$$

Now, one number is 280

$$\therefore 280 \times \text{Other number} = 40 \times 560$$

$$\Rightarrow \text{Other number} = \frac{40 \times 560}{280} = 80$$

6. Show that any positive odd integer is of the form  $6q+1$ ,  $6q+3$  or  $6q+5$  where  $q$  is some integer

Apply Euclid's Division lemma to  $a$  and 6, we have

$$a = 6q + r \text{ where } 0 \leq r < 6$$

Thus,  $r$  can take values 0, 1, 2, 3, 4, 5.

$$\text{Consider the equation, } a = 3q + r$$

**Case 1:** When  $r = 0$

$$\text{Thus, } a = 6q$$

Rewriting the above equation, we have

$$a = 2(3q)$$

Which is an even number.

**Case 2:** When  $r = 1$

$$\text{Thus, } a = 6q + 1$$

Rewriting the above equation, we have

$$a = 2 \times 3q + 1$$

$$= 2m + 1, \text{ where } m = 3q$$

Which is an odd number.

**Case 3:** When  $r = 2$

Thus,  $a = 6q + 2$

Rewriting the above equation, we have

$$a = 2x(3q + 1)$$

$$= 2m, \text{ where } m = 3q + 1$$

Which is an even number.

**Case 4:** When  $r = 3$

Thus,  $a = 6q + 3$

Rewriting the above equation, we have

$$a = 2x(3q + 1) + 1$$

$$= 2m + 1, \text{ where } m = 3q + 1$$

Which is an odd number.

**Case 5:** When  $r = 4$

Thus,  $a = 6q + 4$

Rewriting the above equation, we have

$$a = 2x(3q + 2)$$

$$= 2m, \text{ where } m = 3q + 2$$

Which is an even number.

**Case 6:** When  $r = 5$

Thus,  $a = 6q + 5$

Rewriting the above equation, we have

$$a = 2x(3q + 2) + 1$$

$$= 2m + 1, \text{ where } m = 3q + 2$$

Which is an odd number.

Therefore, any positive odd integer is of the form

$6q + 1$ ,  $6q + 3$  or  $6q + 5$ , where  $q$  is some integer.

**7. Using Euclid's division algorithm, find HCF of 56, 96 and 404.**

Applying Euclid's division algorithm to 56 and 96

$$96 = 56 \times 1 + 40$$

$$56 = 40 \times 1 + 16$$

$$40 = 16 \times 2 + 8$$

$$16 = 8 \times 2 + 0$$

$$\therefore \text{HCF}(56, 96) = 8$$

Next, apply Euclid's division algorithm to 8 and 404

$$404 = 8 \times 50 + 4$$

$$8 = 4 \times 2 + 0$$

Thus,  $\text{HCF}(56, 96, 404) = 4$

**8. Find HCF and LCM of 404 and 96 and verify that  $\text{HCF} \times \text{LCM} = \text{Product of the two given numbers}$ .**

$$404 = 2 \times 2 \times 101 = 2^2 \times 101$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$$

$$\therefore \text{HCF of } 404 \text{ and } 96 = 2^2 = 4$$

[Product of smallest power of each common factor]

$$\text{LCM of } 404 \text{ and } 96 = 101 \times 2^5 \times 3 = 9696$$

[Product of greatest power of each prime factor]

$$\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$$

$$\text{Also } 404 \times 96 = 38784$$

Hence  $\text{HCF} \times \text{LCM} = \text{Product of } 404 \text{ and } 96$ .

**9. Express  $5.\overline{4178}$  in the  $\frac{p}{q}$  form**

$$\text{Let } x = 5.\overline{4178}$$

$$\text{Or } x$$

$$\text{Thus, } 5.\overline{4178} = \frac{27062}{4995}$$

**10. Find the LCM and HCF of 1296 and 5040 by prime factorisation method:**

$\therefore$

2	5040
2	2520
2	1260
2	630
3	315
3	105
5	35
	7

and

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
	3

$$\begin{aligned} \therefore 5040 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \\ &= 2^4 \times 3^2 \times 5 \times 7 \end{aligned}$$

$$\begin{aligned} 1296 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \\ &= 2^4 \times 3^4 \end{aligned}$$

$$\begin{aligned} \therefore \text{LCM} &= \text{Product of each prime factor with highest powers} \\ &= 2^4 \times 3^4 \times 5 \times 7 \\ &= 16 \times 81 \times 5 \times 7 = 45360 \end{aligned}$$

$$\begin{aligned} \text{HCF} &= \text{Product of common prime factors with lowest powers} \\ &= 2^4 \times 3^2 \\ &= 16 \times 9 = 144 \end{aligned}$$

11. The decimal expansion of the rational number  $\frac{43}{2^4 \times 5^3}$ , Will terminate after how many places of decimal?

$$\begin{aligned} \frac{43}{2^4 \times 5^3} &= \frac{43 \times 5^1}{2^4 \times 5^3 \times 5^1} \\ &= \frac{43 \times 5}{2^4 \times 5^3 \times 5^1} \end{aligned}$$

Thus, will terminate after 4 places of Thus,  $2^4 \times 5$  decimal

12. Show that any positive odd integer is of the form  $4q + 1$  or  $4q + 3$  where  $q$  is a positive integer.

Let  $a$  be any positive integer and  $b = 4$ . Applying Euclid's division lemma there exist integers  $q$  and  $r$  such that

$$a = 4q + r,$$

where  $0 \leq r < 4$   $a = 4q$  or  $4q + 1$  or  $4q + 2$  or  $4q + 3$

However since ' $a$ ' is odd we reject the case  $4q$  and  $4q+2$  as they both are divisible by 2.

Therefore, any positive odd integer is of the form  $4q + 1$  or  $4q + 3$ .

13. Check whether  $14^n$  can end with the digit zero for any natural number  $n$  ?

Let, us suppose that  $14^n$  ends with the digit 0 for some  $n \in \mathbb{N}$   $14^n$  is divisible by 5

But, prime factors of 14 are 2 and 7.

$\therefore$  Prime factor of  $(14)^n$  are  $(2 \times 7)^n$

It is clear that in prime factorisation of  $14^n$  there is no place for 5.

$\therefore$  By Fundamental theorem of Arithmetic.

Every composite no. can be expressed as a product of primes and this factorisation is unique a part from the order in which the prime factor occur. Our Supposition is wrong. Hence, there exists no natural number  $n$  for which  $14^n$  ends with the digit zero.

**14. Prove that  $\sqrt{2}$  is an irrational number.**

Let, if possible to the contrary that  $\sqrt{2}$  is not irrational number i.e., 2 is a rational number. That mean  $\sqrt{2}$  can be expressed in  $\frac{p}{q}$  form where  $p$  and  $q$  are coprime positive integers

and  $q \neq 0$ . So  $\sqrt{2} = \frac{p}{q}$

$$\Rightarrow p^2 = 2 q^2$$

Thus,  $p^2$  is a multiple of 2

$\Rightarrow p$  is a multiple of 2.

Let  $p = 2m$  for some integer  $m$ .

$$\Rightarrow p^2 = 2 q^2$$

Thus,  $q^2$  is a multiple of 2.

$\Rightarrow q$  is a multiple of 2.

Hence, 2 is a common factor of  $p$  and  $q$ . This contradicts the fact that  $p$  and  $q$  are coprimes.

$\therefore$  Our supposition is wrong.

Hence,  $\sqrt{2}$  is an irrational number.

**I. Long answer choice questions**

**1. Prove that one of every three consecutive positive integers is divisible by 3.**

Let  $n$  be any positive integer.

$$\therefore n = 3q + r, \text{ where } r = 0, 1, 2$$

Putting  $r = 0$ ,

$$n = 3q + 0 = 3q,$$

which is divisible by 3.



**2. State Fundamental theorem of Arithmetic. Find LCM of numbers 2520 and 10530 by prime factorization method.**

**Fundamental theorem of arithmetic:** Every composite number can be expressed as the product of powers of primes and this factorization is unique.

$$\text{Since, } 2520 = 2^3 \times 3^2 \times 5 \times 7$$

$$\text{and } 10530 = 2 \times 3^4 \times 5 \times 13$$

$$\text{LCM} = 2^3 \times 3^4 \times 5 \times 7 \times 13$$

$$= 294840$$

3. A fruit vendor has 990 apples and 945 oranges. He packs them into baskets. Each basket contains only one of the two fruits but in equal number. Find the number of fruits to be put in each basket in order to have minimum number of baskets.

HCF of 990 and 945

$$\begin{array}{r} 945 \overline{) 990} \quad (1 \\ - 945 \\ \hline \end{array}$$

$$\begin{array}{r} 45 \overline{) 945} \quad (21 \\ - 90 \\ \hline \end{array}$$

$$- 90$$

$$45$$

$$- 45$$

$$0$$

$$990 = 945 \times 1 + 45$$

$$945 = 45 \times 21 + 0$$

Since, HCF of 990 and 945 is 45.

Thus, the fruit vendor should put 45 fruits in each basket to have minimum number of baskets.

4. Can the number  $6^n$ ,  $n$  being a natural number, end with the digit 5? Give reasons

If  $6^n$  ends with 0 or 5, then it must have 5 as a factor.

But only prime factors of  $6^n$  are 2 and 3.

$$\therefore 6^n = (2 \times 3)^n = 2^n \times 3^n$$

From the fundamental theorem of arithmetic, the prime factorization of every composite number is unique.

$\therefore 6^n$  can never end with 0 or 5.

5. For any positive integer  $n$ , prove that  $n^3 - n$  is divisible by 6

$$n^3 - n = n(n^2 - 1)$$

$$= n(n-1)(n+1)$$

$$\text{Let } a = 3q + r$$

$$\text{where } r = 0, 1, 2$$

**Case I:** When  $r = 0, a = 3q$

$$\begin{aligned} n^3 - n &= n(n-1)(n+1) \\ &= 3q(3q-1)(3q+1) \\ &= 3m \end{aligned}$$

where  $m = q(3q-1)(3q+1)$

$(n^3 - n)$  is divisible by 3

**Case II:** When  $r = 1, a = 3q + 1$

$$\begin{aligned} n^3 - n &= n(n-1)(n+1) \\ &= (3q+1)(3q)(3q+2) \\ &= (3q)(3q+1)(3q+2) \\ &= 3m \end{aligned}$$

Where  $m = q(3q+1)(3q+2)$

$\therefore (n^3 - n)$  is divisible by 3

**Case III:** When  $r = 2, a = 3q + 2$

$$\begin{aligned} n^3 - n &= n(n-1)(n+1) \\ &= (3q+2)(3q+1)(3q+3) \\ &= 3(q+1)(3q+2)(3q+1) \\ &= 3m \end{aligned}$$

Where  $m = (q+1)(3q+2)(3q+1)$

$\therefore (n^3 - n)$  is divisible by 3

Let  $a = 2q + r$

where  $r = 0, 1$

**Case I:** When  $r = 0$

$$\begin{aligned} n^3 - n &= n(n-1)(n+1) \\ &= 2q(2q-1)(2q+1) \\ &= 2m \end{aligned}$$

Where  $m = q(2q-1)(2q+1)$

$\therefore (n^3 - n)$  is divisible by 3

**Case II:** When  $r = 1, a = 2q + 1$

$$\begin{aligned} n^3 - n &= n(n-1)(n+1) \\ &= 2q(2q+1)(2q+2) \\ &= 2m \end{aligned}$$

where  $m = q(2q + 1)(2q + 2)$

So we can say that one of the numbers among  $n, (n - 1)$  and  $(n - 1)$  is always divisible by 2 and 3

∴ As per the divisibility rule of 6

The given number is divisible by 6

∴  $n^3 - n$  is divisible by 6 Hence Proved

**6. Find the LCM of 205, 0.5 and 0.175**

$$\text{LCM Of Rational} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

Number are  $\frac{25}{10}, \frac{5}{10}, \frac{175}{1000}$

Now  $25 = 5 \times 5$ ;  $5 = 5 \times 1$ ;  $175 = 5 \times 5 \times 7$

∴ LCM of (25, 5, 175)  $5 \times 5 \times 7 = 175$

Also  $10 = 2 \times 5$ ;  $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

∴ HCF of (10, 10, 1000) = 10

∴ LCM of (2.5, 0.5, 0.175)  $= \frac{175}{10} = 17.5$

**7. A forester wants to plant 66 apple trees, 88 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also he wants to make distinct rows of trees (i.e., only one type of trees in one row). Find the number of minimum rows required .**

$66 = 2 \times 3 \times 11$ ;  $88 = 2 \times 2 \times 2 \times 11$ ;  $110 = 2 \times 5 \times 11$

∴ HCF of 66, 88 and 110 = 22

∴ Number of trees in each row = 22

∴ Number of rows  $= \frac{66}{22} + \frac{88}{22} + \frac{110}{22} = 3 + 4 + 5 = 12$

**8. The HCF of 2472, 1284 and a third number N is 12, If their LCM is  $2^3 \times 3^3 \times 5 \times 103 \times 107$ . Then find the number N.**

$2472 = 2^3 \times 3 \times 103$

$1284 = 2^3 \times 3 \times 107$

∴ LCM  $= 2^3 \times 3^2 \times 5 \times 103 \times 107$

∴  $N = 2^3 \times 3^2 \times 5 = 180$

**9. Prove that  $15 + 17\sqrt{3}$  be an irrational number**

Let  $\sqrt{3} = \frac{a}{b}$  where  $a$  and  $b$  are coprime integers  $b \neq 0$

Squaring both sides, we get  $3 = \frac{a^2}{b^2}$

Multiplying with  $b$  on both sides, we get

$$3b = \frac{a^2}{b}$$

$$\text{LHS} = 3 \times b = \text{Integer}$$

$$\text{RHS} = \frac{a^2}{b} = \frac{\text{Integer}}{\text{Integer}} = \text{Rational number}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$  Our supposition is wrong

$\Rightarrow \sqrt{3}$  is irrational

Let  $15 + 17\sqrt{3}$  be a rational number

$$15 + 17\sqrt{3} = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are coprime}$$

$$b \neq 0$$

$$\Rightarrow 17\sqrt{3} = \frac{a}{b} - 15$$

$$\sqrt{3} = \frac{a-15b}{17b}$$

$\frac{a-15b}{17b}$  is a rational number

But  $\sqrt{3}$  is irrational

$$\therefore \sqrt{3} = \frac{a-15b}{17b}$$

**III. Long answer choice questions**

1. A, B and C starts cycling around a circular path in the same direction at the same time  
Circumference of the path is 1980 m. If speed of A is 330 m/min, speed of B is 198 m/min  
and that of C is 220 m/min and they start from the same point, then after what time will they  
be together at the starting point?

$$\text{As, Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time taken by A to complete one round

$$= \frac{1980}{330} = 6 \text{ min}$$

Time taken by B to complete one round

$$= \frac{1980}{198} = 10 \text{ min}$$

Time taken by C to complete one round

$$= \frac{1980}{220} = 9 \text{ min}$$

The three cyclists will be together after LCM (6, 10, 9)

$$6 = 2 \times 3$$

$$10 = 2 \times 5$$

$$9 = 3^2$$

$$\text{LCM}(6, 10, 9) = 2^1 \times 3^2 \times 5 = 90 \text{ min.}$$

2. The HCF of 408 and 1032 is expressible in the form  $1032m - 2040$ . Find the value of  $m$ . Also, find the LCM of 408 and 1032.

Let us find HCF of 408 and 1032

Here,  $1032 > 408$

$$\therefore 1032 = 2 \times 408 + 216$$

$$408 = 1 \times 216 + 192$$

$$216 = 1 \times 192 + 24$$

$$192 = 8 \times 24 + 0$$

Thus, HCF of 408 and 1032 is 24.

Now, HCF (408, 1032)

$$\text{i.e. } 24 = 1032x - 2040$$

$$\Rightarrow 1032x = 24 + 2040$$

$$\Rightarrow 1032x = 2064$$

$$\Rightarrow x = \frac{2064}{1032} = 2$$

$$\text{Again, } 408 = 2^3 \times 3 \times 17$$

$$1032 = 2^3 \times 3 \times 43$$

$$\therefore \text{LCM of } 408 \text{ and } 1032 = 2^3 \times 3 \times 17 \times 43$$

$$1032 = 2^3 \times 3 \times 43$$

$$\therefore \text{LCM of } 408 \text{ and } 1032 = 2^3 \times 3 \times 17 \times 43 = 17544$$

Next Generation School

3. Obtain the HCF of 420 and 272 by using Euclid's division algorithm and verify the same by using Fundamental theorem of Arithmetic.

**Case I:** Using Euclid's division algorithm

$$420 = 272 \times 1 + 148$$

$$272 = 148 \times 1 + 124$$

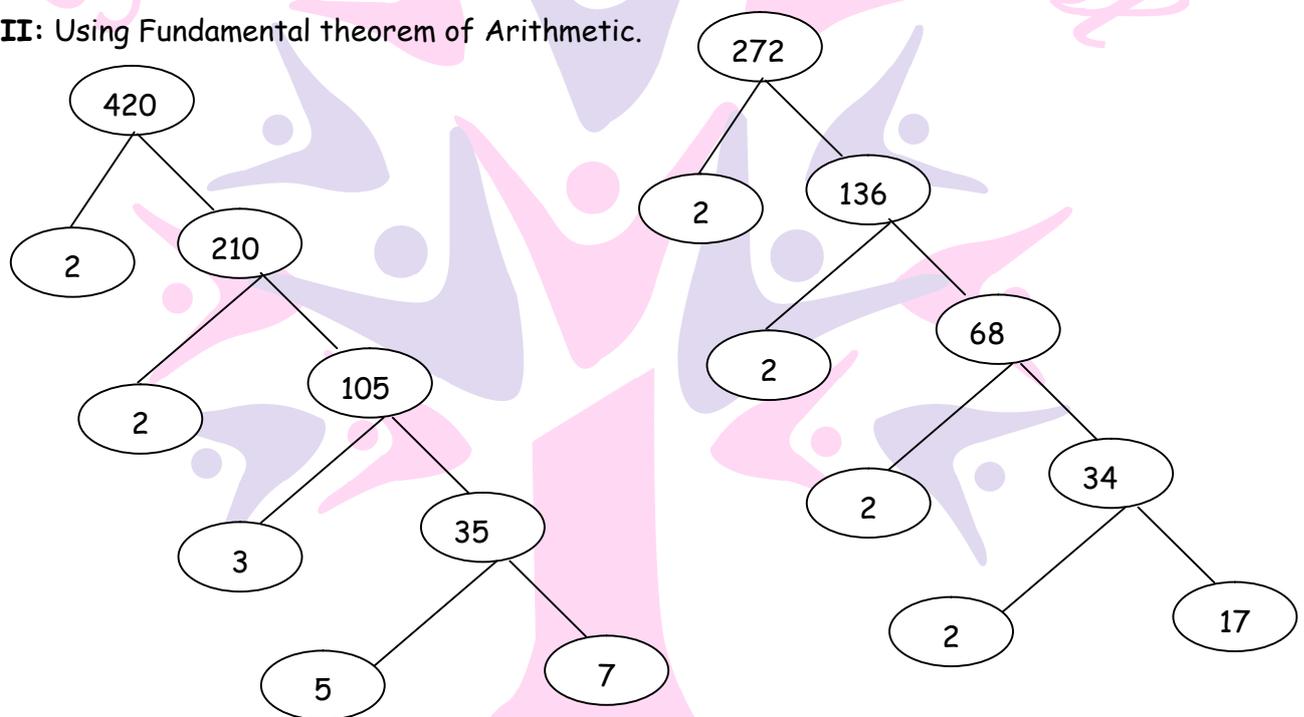
$$148 = 124 \times 1 + 24$$

$$124 = 24 \times 5 + 4$$

$$24 = 4 \times 6 + 0$$

$$\therefore \text{HCF}(420, 272) = 4$$

**Case II:** Using Fundamental theorem of Arithmetic.



$$420 = 2 \times 2 \times 3 \times 5 \times 7$$

$$= 2^2 \times 3 \times 5 \times 7$$

$$\therefore \text{HCF}(420, 272) = 2^2 \times 4$$

$$272 = 2 \times 2 \times 2 \times 2 \times 17$$

$$= 2^4 \times 17$$

From I and II the result obtained is verified.

4. If 'h' is HCF of 609 and 957. Find x and y satisfying  $h = 609x + 957y$ . Also show that x and y are not Here  $957 > 609$

So, starting with

a = 957 and b = 609 and applying Euclid's division algorithm

we get

$$957 = 609 \times 1 + 348 \quad \dots(i)$$

$$609 = 348 \times 1 + 261 \quad \dots(ii)$$

$$348 = 261 \times 1 + 87 \quad \dots(iii)$$

$$261 = 87 \times 3 + 0 \quad \leftarrow \text{remainder}(r) \quad \dots(iv)$$

$$\Rightarrow \text{HCF}(609, 957) = 87$$

Alternatively,  $609 \overline{)957} (1$

$$\begin{array}{r} 609 \\ \underline{348} 609 (1 \\ \underline{261} 348 (1 \\ \underline{261} 261 (3 \\ \underline{261} 0 \end{array}$$

From (iii)  $87 = 348 - 261 \times 1$

$$= 348 - (609 - 348 \times 1) \times 1$$

$$[\because \text{From (ii) } 261 = 609 - 348 \times 1]$$

$$= 348 - 609 \times 1 + 348 \times 1$$

$$= 348 \times 2 - 609 \times 1$$

$$= (957 - 609 \times 1) \times 2 - 609 \times 1$$

$$= 957 \times 2 - 609 \times 2 - 609 \times 1$$

$$= 957 \times (2) + 609 \times (-2 - 1)$$

$$= 957 \times (2) + 609 \times (-3)$$

...(iv)

$$\text{Thus, } 87 = 957y + 609x$$

$$\text{where } y = 2$$

$$\text{and } x = -3$$

Adding and subtracting  $957 \times 609$  in (iv),

we get

$$87 = 957 \times 2 + 609 \times (-3) + 957 \times 609 - 957 \times 609$$

$$= 957 \times 2 + 957 \times 609 + 609 \times (-3) + 609 \times (-957)$$

$$= 957 \times (2 + 609) + 609 \times (-3 - 957)$$

$$\Rightarrow 87 = 957 \times (611) + 609 \times (-960)$$

$$= 957x + 609y$$

Where  $x = 611$ ,  $y = 960$

(v) represents another linear combination and hence  $x$  and  $y$  are not unique.

