Grade X

Lesson : 2 polynomials

Objective Type Questions

I. Multiple choice questions

1. Sum of zeroes of polynomial ax^2+bx + c is zero then value of 'b' is

a) -1 **b) 0** c) 1 d) $\frac{c}{a}$

2. If product of zeroes of polynomial (ax^2+bx + c) is zero then value of 'c' is

a) 3 b) 2 c) 1 **d) 0**

3. Find $(\alpha + \beta) + \alpha\,\beta$ for polynomial x^2- 4x + 3

a) 3 b) 7 **c) 7** d) 12

4. For Q No.3 Find value of $\frac{1}{\alpha} + \frac{1}{\beta}$

a) 3 b) 4 c) 1 **d) $\frac{4}{3}$**

5. Number of real zeroes of polynomial $(x^2 + 1)$ is

a) 1 b) 2

c) 3 **d) No real zero is possible**

6. If one of the zeroes of polynomial x^2+ kx – 8 is zero then value of 'k' is

a) 0 b) 8

c) -8 **d) No real value of 'k' exist**

7. Polynomial whose zeroes are 1 and 2 is

a) x^2- 3x + 2 b) x^2+ 3x + 2 c) x^2+ 3x - 2 d) x^2- 3x – 2

8. For what value of 'k' given polynomial (k-1) x^2+ x + 6 is a quadratic polynomial?

a) 1 b) -1 c) 0 **d) k≠1**

9. If zeroes of polynomial are 2 and 3, then polynomial is

a) x^2- 6 b) x^2+ 5x + 6

c) x^2- 5x + 6 d) None of these

10. Degree of zero polynomial is

a) 0 b) 1 c) ∞ **d) Not defined**

11. If (-4) is the zero of a polynomial x^2- x – (2+2k) then value of 'k' is

a) 1 b) 2 **c) 9** d) 12

12. Degree of constant polynomial is

a) 0 b) constant c) 1 d) None of these

13. Find value of $(\alpha - \beta)$ for polynomial x^2- $7x$ + 2=

a) 1 b) -1

c) both (a) and (b) d) None of these

14. If zeroes of polynomial ax^2+bx + c are α *and* β then zeroes of polynomial k x (ax^2+bx + c) are

a) 2 α *and* 2 β b) $k\alpha$ *and* $k\beta$ c) $\frac{\alpha}{k}$ and $\frac{\beta}{k}$ **d) $\alpha + \beta$**

15. Zeroes of polynomial ($\sqrt{3}x^2$-8x + 4 $\sqrt{3}$)

a) $2\sqrt{3}$ and $\frac{2}{\sqrt{3}}$ b) $2\sqrt{3}$ and $\frac{-2}{\sqrt{3}}$

c) -$2\sqrt{3}$ and $\frac{2}{\sqrt{3}}$ d) -$2\sqrt{3}$ and $\frac{-2}{\sqrt{3}}$

16. If zeroes of polynomial are $4x^2$-2 x +(p-4) are reciprocal of each other then value of 'p' is

a) 8 b) 6 c) 4 d) 2

17. Which of the following cannot be the remainder, if p(x) a polynomial of degree ≥ 2 is divided by (x+1)?

a) x-1 b) x+1 c) x **d) All of these**

18. If one of the zeroes of polynomial $3x^2$- $8x$ + (2k-1) is seven times of other then value of 'k' is

a) $\frac{1}{3}$ b) $\frac{7}{3}$ **c) $\frac{2}{3}$** d) 0

19. If α and β are zeroes of polynomial x^2- 5x+k such that α - β = 1 then value of 'k' is

a) 1 b) 2 c) 3 **d) 6**

20. If α and β are zeroes of polynomial x^2-6x+a such that 3α + 2β=20 then value of 'a' is

a) 0 b) 8 **c) -16** d) 16

21. α , β and γ are zeroes of polynomial x^3+9, then value of $\alpha^{-1} + \beta^{-1} + \frac{1}{\gamma}$ is

a) $\frac{-1}{3}$ **b)** $\frac{-2}{3}$ c) $\frac{-4}{3}$ d) None of these

22. If zeroes of polynomial are 1 and -2 then polynomial is

a) x^2-1 b) x^2-4 **c) $x^2 + x$-2** d) $x^2 + x$+2

23. If one of zeroes of polynomial $x^3 + a\,x^2$+bx+c is (-1) then product of zeroes is

a) 1-a+b b) a + b c) a-b d) 1+a+b

24. If zeroes of ax^2+bx+c are reciprocal of each other then

a) c = 0 b) $\frac{c}{a}$ = 1 c) c-a = 0 **d) All of these**

25. In polynomial $a\,x^2$+bx+c if both zeroes are positive then sign of 'b' is

a) positive **b) negative**

c) both a and b d) b=0, not positive, not negative

26. If zeroes of polynomial ax^2+bx+c are equal the

a) c and a have same sign b) c and a have opposite sign

c) one of c and a must be zero d) None of these

27. If one of zeroes of cubic polynomial $a\,x^3$+bx^2+cx + d is zero the value of their product is

a) b b) c c) d **d) 0**

28. If one of the zeroes of polynomial x^2+x+k is zero then value of their product is

a) b b) c c) d **d) 0**

29. For real zeroes of polynomial ax^2+bx+c which condition is true

a) b^2-4ac=0 b) b^2-4ac>0

c) both (a) and (b) d) None of these

30. If α and β are zeroes of polynomial ax^2+bx+c, which condition is true

a) 7 b) -12 **c)** $\frac{7}{12}$ d) None of these

31. If one of zeroes of polynomial ax^2+bx+c is zeroes then value of 'c' is

a) 1　　b) $\frac{b}{a}$　　c) $\frac{b}{a}$　　**d) 0**

32. If α and β are zeroes of polynomial ax^2+bx+c, then polynomial whose zeroes $\frac{\alpha}{k}$ and $\frac{\beta}{k}$ are

a) k (x^2+bx+c)　　b) $\frac{1}{k}$ (ax^2+bx+c)　　c) (ax^2+bx-c)　　**d) (ax^2+bx+c)**

33. If α and β are zeroes of polynomial ax^2+bx+c, if zeroes are equal in magnitude and opposite in sign then value of 'b' is

a) 1　　b)-1　　**c) 0**　　d) ± 1

34. If (x- α) (x- β) = (x^2-5x-6) then α is the zero and β is negative zero then value of $\frac{\alpha}{\beta}$ is

a) -6　　b) +6　　c) $\frac{3}{2}$　　d) $\frac{2}{3}$

35. If α and β are zeroes of polynomial x^2-p($x+1$)-c, then value of (1+ α) (1+ β) is

a) 1　　**b) 1-c**　　c) 1+c　　d) c

36. Number required to subtract from polynomial x^2-16x+30, so that one of the zero becomes

a) 5　　b) 10　　c) 12　　**d) 15**

37. If polynomial x^2+bx+c has no real zero then which is correct?

a) b^2 = 4c　　b) b^2- 4ac is negative

c) (b^2 - 4c) < 0　　d) b^2-4$ac > 0$

38. Graph of polynomial x^2-7x-8 but x axis at

a) (8,-1)　　b) (-8,+1)　　**c) (-8,-1)**　　d) (8, 1)

39. If both zeroes of polynomial are equal then its graph meet x- axis at

a) touch only at one point　　b) meet at two points

c) cannot meet x – axis　　d) None of these

40. In a quadratic polynomial ax^2+bx+c, which is always true?

a) a≠0　　b) b= 0　　c) c =0　　d) None of these

II. Multiple choice questions

1. Which of the following is not a polynomial.

a) $3x + 5$ b) $3y^3$-$4y^3$ + 2y

c) x^2-3 **d)** $\mathbf{\frac{1}{x+2}}$

2. If 2 is a zero of polynomial f(x) = ax^2-3(a-1) x-1, then the value of a is.

a) 0 b) 2 **c)** $\mathbf{\frac{5}{2}}$ d) $\frac{1}{2}$

3. One of the zeroes of the quadratic polynomial (k-1) x^2+3x+k is 2 then the value of k is

a) $\mathbf{\frac{4}{3}}$ b) $\frac{4}{3}$ c) $\frac{2}{3}$ d) $\frac{-2}{3}$

4. If one of the zeroes of the quadratic polynomial x^2 +3 x +k is 2 then the value of k is

a) 10 **b) -10** c) -7 d) 0 -2

5. If 2 and 3 are zeroes of the polynomial $3x^2$ -2k x +2m, then the values of k and m are

a) m = $\frac{9}{2}$ and k = 15 b) m = $\frac{15}{2}$ and k = 9

c) m = 9 and k = $\mathbf{\frac{15}{2}}$ d) m = 15 and k = 9

6. The value of p, for which (-4) is a zero of the polynomial x^2 -2 x – (7 p+3) is ______.

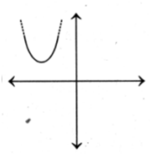
a) 3 b) 2 c) 4 d) -2

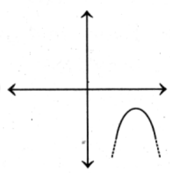
7. If the graph of a polynomial intersects the x - axis at only one point, it can be a

a) linear b) quadratic

c) cube d) None of these

8. Which of the following is not the graph of a quadratic polynomial?


a)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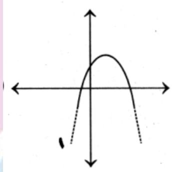
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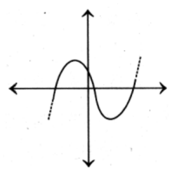

d)

9. The graph of a quadratic polynomial is ______.

a) straight the **b) Parabola**

c) hyperbola d) None of these

10. If one zero of polynomial x^2 -kx +1 is 2 + $\sqrt{3}$ then the zero will be ______.

a) -2 + $\sqrt{3}$ b) -$\sqrt{3}$ -2 **c) 2 + $\sqrt{3}$** d) $\sqrt{3}$ +1

11. The zeros of the quadratic polynomial $(x^2$ +5 x +6) are

a) -2 and -3 b) 3 and 4 c) 3 and 2 d) 2 and 1

12. Zeroes of P (z) = z^2 - 27 are ______.

a) $\pm$ 3 $\sqrt{3}$ b) + 3 c) +9 d) + $\sqrt{3}$ and $-\sqrt{3}$

13. The zeroes of the quadratic polynomial

f(x) = abx^2 + $b^2 - ac)$ x-bc are

a) $\frac{b}{c}$ and $\frac{c}{b}$ b) $\frac{a}{c}$ and $\frac{a}{b}$ **c) $\frac{-b}{c}$ and $\frac{c}{b}$** d)) $\frac{b}{a}$ and $\frac{-c}{b}$

14. The number of polynomials having zeroes as -2 and 5 is

a) 1 b) 2 c) 3 **d) more than 3**

15. 1 and 2are the zeroes the polynomial x^2 -3x +2

a) True b) False c) Can't say d) partially True / False

16. Every real number is the zeroes of zero polynomial

a) True b) False

c) Can't say d) partially True / False

17. p(x) = x -1 and g(x) = x^2 -2x +1 p(x) is a factor of g(x)

a) True b) False

c) Can't say d) partially True / False

18. The value of k for which 3 is a zero of polynomial $2x^2$ + x + k is _________.

a) 21 b) 20 **c) -21** d) 18

19. If zeroes $\propto$ *and* β of a polynomial x^2 -7x + k are such that $\propto$-$\beta = 1,$

then the value of k is

a) 21 **b) 12** c) 9 d) 8

20. Sum of zeroes of the quadratic polynomial = $\frac{Co\ efficient\ of\ x}{Co\ efficient\ of\ x^2}$

a) true b) false

c) Can't say d) partially True / False

21. If $\propto$ *and* β are zeroes of the quadratic polynomial x^2 -6x + a the value of 'a' if 3$\propto$ + 2β =20 is -12

a) true **b) false** c) Can't say d) partially True / False

22. If one of the zeroes of a quadratic polynomial of the form x^2 +ax + b is the negative of the other, then it.

a) has no linear term and the constant term is negative

b) has no linear term and the constant term is positive

c) can have a linear term but the constant term is negative

d) can have a linear term but the constant term is positive

23. If p and q are zeroes of $3x^2$ + $2x$ -9 then value of p-q is

a) -3 b) $\frac{2}{3}$ **c) 1** d) 20

24. The polynomial whose zeroes are ($\sqrt{2}$+1) and ($\sqrt{2}$-1) is

a) x^2 +2$\sqrt{2}x$ + 1 **b) x^2 -2$\sqrt{2}x$ + 1** c) x^2 -2$\sqrt{2}x$ – 1 d) x^2 -2$\sqrt{2}x$ – 1

25. If $\propto$ *and* β are the zeroes of the polynomial $2y^2$ + 7y +5, then the value of $\propto + \beta + \propto\beta$ is

a) -1 b) 0 c) 1 d) 2

26. If $\propto$ *and* β are the zeroes of the quadratic polynomial $f(x) = 3x^2$ – 5x – 2 then $\propto^3$ + β^3 is equal to

a) $\frac{215}{27}$ b) $\frac{357}{21}$ c) $\frac{115}{28}$ d) $\frac{325}{31}$

27. If $\propto$ *and* β are the zeroes of $4x^2$ + $3x$ + 7, then the value of $\frac{1}{\propto}$ + $\frac{1}{\beta}$ is

a) -$\frac{8}{7}$ **b) $-\frac{3}{7}$** c) $\frac{2}{7}$ d) $\frac{6}{8}$

28. If the zeroes of the quadratic polynomial x^2 +(a + 1) x + b are 2 and-3 then

a) a = -7 b = -1 b) a = 5 b =-1 c) a = 2 b =-6 **d) a=0 b=-6**

29. If the sum and difference of zeroes of quadratic polynomial are -3 and -19 respectively. Then the difference of the squares of zeroes is

a) 20 **b) 30** c) 15 d) 25

30. If sum and product of zeroes of quadratic polynomial are respectively 8 and 12, then their zeroes are

a) 2 and 6 b) 3 and 4 c) 2 and 8 d) 2 and 5

31. If m and n are the zeroes of the polynomial $3x^2$ + $11x$-4 then find the value of $\frac{m}{n}$ + $\frac{n}{m}$

a) $\frac{145}{12}$ **b) - $\frac{145}{2}$** c) $\frac{145}{7}$ d) $\frac{-145}{15}$

32. If one zero of the polynomial $3x^2$ -8x+ 2k + 1 is seven times the other, then the value of k is.

a) $\frac{2}{5}$ **b) $\frac{2}{3}$** c) $\frac{2}{7}$ d) $\frac{3}{2}$

33. If sum of the squares of zeroes of the quadratic polynomial f(x) = x^2 -4x + k is 30 , then the value of k is

a) -2 b) -3 c) -4 d) 2

34. If $\propto and\ \beta$ are the zeroes of the polynomial x^2 –p(x +1) +c such that ($\propto +1$) ($\beta + 1$)= 0, then the value of c is

a) -2 b) 2 **c) -1** d) 1

35. The value of k such that the polynomial x^2 –(k+6) x+2 (2k-1) has sum of its zeroes equal to half of their product is

a) -4 b) 4 c) -7 **d) 7**

36. The sum and the product of zeroes of the polynomial f(x) = 4 x^2 -27x + 3 k^2 are equal the value of k is

a) k=3 b) k=-3 **c) k$\pm$3** d) k =2

37. If $\propto and\ \beta$ are the zeroes of the quadratic polynomial f(x) = x^2 -4 x + 3, then the value of $\propto^4$ β^3 + $\propto^3$ β^4 is

a) 104 **b) 108** c) 112 d) 5

38. The quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is

a) x^2 +5 x + 6 b) x^2 - 5 x + 6 c) x^2 -5 x -6 d) -x^2 +5 x + 6

39. The quadratic polynomial whose zeroes are 2$\sqrt{7}$ and 5$\sqrt{7}$ is

a) x^2 -3$\sqrt{7}$ x -70 b) x^2 +3$\sqrt{7}$ x + 70 **c) x^2 +3$\sqrt{7}$ x -70** d) x^2 -3$\sqrt{7}$ x + 70

40. The quadratic polynomial whose zeroes are $3+\sqrt{2}$ and $3-\sqrt{2}$, is

a) x^2 -3 x + 5 **b) x^2 -6 x + 7** c) x^2 -7 x +6 d) -x^2 +8 x + 12

41. If $\propto$ *and* β are the zeroes of a quadratic polynomial f(x) = x^2 + x -2, then the quadratic polynomial whose zeroes are $2\propto +1$ and $2\beta + 1$ is

a) x^2 + 9 b) x^2 - 4 **c) x^2 - 9** d) x^2 + 4

42. If $\propto$ *and* β are the zeroes of a quadratic polynomial x^2 - 5, then the quadratic polynomial whose zeroes are 1 + $\propto$ and 1 + β is

a) x^2 +2 x + 24 **b) x^2 -2 x - 24** c) x^2 -2 x + 24 d) None of these

43. The number of value of k for which the quadratic polynomial whose kx^2 + x + k has equal zeroes is

a) 4 b) 1 **c) 2** d) 3

44. p(x) = $5x^2$ + $3x^2$+ 7 x +2 then match the value of Column I with that of Column II

	Column I		**Column I**
A	p (1)	P.	2
B.	P (2)	Q.	11
C.	P (5)	R.	-13
D.	P (-1)	S.	-64
E.	P (-2)	T.	587

A B C D E — A B C D E

a) O T R P S — b) O R T S P

c) O P T R S — d) T R O S P

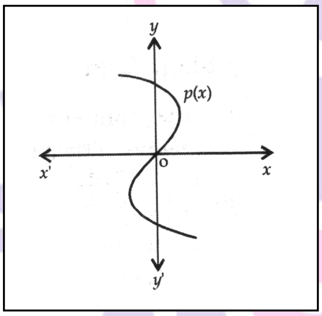
45. If $\propto$ *and* β are the zeroes of the polynomial $2x^2$- 4 x + 5 then match the value of Column I with that of Column II

	Column I		**Column I**
A	$\frac{1}{\propto} + \frac{1}{\beta}$	1.	-6
B.	$(\propto - \beta)^2$	2.	$\frac{-4}{25}$
C.	$\frac{1}{\propto^2} + \frac{1}{\beta^2}$	3.	$\frac{-2}{5}$
D.	$\frac{\propto}{\beta} + \frac{\propto}{\beta}$	4.	$\frac{4}{25}$

	A	B	C	D		A	B	C	D
a)	**4**	**1**	**2**	**3**	b)	4	2	1	3
b)	1	2	3	4	b)	1	4	2	3

III. Multiple choice questions

1. The graph of p(r) is shown alongside. The number of zeroes of p(r) are:



(a) 1 (b) 2 (c) 3 (d) 4

2. If one zero of polynomial $(k^2 + 16)\,x^2 + 13x + 8k$ is reciprocal of the other then k is equal to

(a) -4 **(b) +4** (c) 8 (d) 2

3. If α and β are zeroes of the polynomial $p(x) = x^2 + mx + n$, then a polynomial whose zeroes are $\frac{1}{\alpha}, \frac{1}{\beta}$ is given by

(a) $nx^2 + mx + 1$ (b) $mx^2 + x + n$

(c) $x^2 + nx + m$ (d) $x^2 - mx + n$

4. If the graph of $y = p(x)$ does not cut the x - axis at any point, then polynomial has

a) one zero b) two zeroes

c) no zeroes d) infinite no. of zeroes

5. Sum of the zeroes of the polynomial $p(x) = -3x^2 + k$ is

a) $\frac{k}{3}$ b) $\frac{-k}{3}$ **c) 0** d) k

6. If -1 is a factor of p(x) = kx^2+ - $\sqrt{2}x$+1, then the value of k is

a) $\sqrt{2}$-1 b) $\sqrt{2}$ + 1 c) -1 -$\sqrt{2}$ d) 1+ $\sqrt{2}$

7. Number of zeroes of a polynomial of degree n is

a) equal to n b) less than n

c) greater than n **d) less than or equal to n**

8. Zeroes of the polynomial p(x)= 2x^2 -9 -3 x are

a) 3, $\frac{3}{2}$ **b) - $\frac{3}{2}$, 3** c) 2, 3 d) 9, $\frac{3}{2}$

9. If $(\alpha - \beta)$, α $(\alpha + \beta)$ are zeroes of the polynomial p(x)= 2x^3 + 16x^2 +15 x -2 value of α is

a) 8 b) 0 c) $\frac{3}{8}$ **d) $\frac{8}{3}$**

10. If n represents number of real zeroes for polynomial ax^3+ bx^2 +c x +d then which of the following inequality is valid

a) $0 < n < 3$ b) $0 \leq n < 3$ **c) $0 < n \leq 3$** d) $0 \leq n \leq 3$

11. Number of quadratic polynomials having -2 and -5 as their two zeroes is:

a) One b) Two c) Three **d) Infinite**

12. If α, β, γ are zeroes of the polynomial p(x) such that $\alpha + \beta + \gamma$ =2, then $\alpha\beta$+ $\beta\,\gamma$ +$\gamma\,\alpha$=5, $\alpha\beta\gamma = -7$ then P(x) is :

(a) x^3 - 2x^2 +5x-7 b) x^3 + 2x^2 $-$5x+7

(c) x^3 - 2x^2 $-$5x-7 **(d) x^3 - 2x^2 +5x+7**

13. If the sum of products of zeroes taken two at a time of polynomial p(x) =x^3 - 5x^2 +c x + 8 is 2 then the value of c is

(a) 2 (b) -2 (c) 8 (d) -5

14. The division algorithm states that given any polynomial p(x) and any non-zero polynomial

g(x) there are polynomial q(x) and r(x) such that p(x) = g(x) + q(x), r(x),where

(a) either = 0 or deg. r(x) $\leq$ deg g(x)

(b) either = 0 or deg. r(x) $>$ deg. g(x)

(c) a linear polynomial or deg. r(x) $=$ deg g(x)

(d) either =0 or deg. r(x) $<$ deg. g(x)

15. If divisor, quotient and remainder are x + 1, 3x -2 and 1 respectively, then dividend is

(a) $3x^2 + x + 1$ (b) $3x^2 - x - 1$ **c) $3x^2 + x - 1$** d) $3x^2 - x + 1$

Fill in the blanks

1. The zeroes of the polynomial x^2- 49 are______.

± 7

2. The quadratic polynomial, whose sum and product of zeroes are 4 and -5 respectively is ______.

$k(x)^2 - 4x - 5$

3. The value of the polynomial p(x) $= 4x^2 - 7$ at x =-2 is ______.

9

4. Product of zeroes of a polynomial p (x) = 6x^2 -7 x -3 is ______.

$-\frac{1}{2}$

5. If one zero of 3x^2- 8 x +2k + 1 is seven times the other, then k is ______.

$\frac{2}{3}$

6. The degree of the constant polynomial is ______.

Zero

7. A real number k is a zero of the polynomial p(x) if and only if ______.

p(k) =0

8. The shape of the graph of a cubic polynomial is ______.

Not fixed

9. If α, ß, and γ are the zeroes of the cubic polynomial is px^3 + qx^2 +r x +s ; a≠0, then

α+ ß+ γ=______.

α ß + ß γ+ γ α =______and α ß γ =______.

$\frac{-q}{p}$, $\frac{r}{p}$, $\frac{-s}{p}$

10. The standard form of the polynomial x^3- x^6+ x^5 + 2x^2- x^4-5 is ______________

$-x^6$+ $x^5$$-x^4$+$x^3$+ 2$x^2$- 5 or – 5+2$x^2$+$x^3$$-x^4$+$x^5$$-x^6$

I Very Short Answer Type Questions 2.1 and 2.2

1. If p(x) is a polynomial of atleast degree one and p(k) = 0, then k is known as

a) value of p (x)

b) zero of p (x)

c) constant term of p(x)

d) none of these

Ans: zero of p(z = x)

Let p(x) = a x + b

Put x = k

P(k) = ak + b = 0

∴ is zero of p(x)

2. If one of the zeroes of the quadratic polynomial (k-1) x^2 +kx +1 is -3then the value of k is

a) $\frac{4}{3}$ b) $\frac{-4}{3}$ c) $\frac{2}{3}$ d) $\frac{-2}{3}$

3. If the zeroes of the quadratic polynomial x^2 + (a+1) x + b are 2 and -3then

a) a = -7, b =-1 b) a = 5, b=-1 c) a=2 **d) a=0 b=-6**

Ans: x^2 + (a+1) x+b

∵ x = 2 is a zero and x = -3 is another zero

∵ $(2)^2 + (a+1)^2$+b =0

and $(-3)^2$ +(a+ 1) (-3) + b = 0

$\Rightarrow$ 4 + 2a +2 +b =0 and 9-3a-3 + b= 0

$\Rightarrow$ 2 a + b =-6 ….910 and -3a+b= -6______(ii)

Solving (i) and (iii) we get 5a = 0

$\Rightarrow$ a = 0 and b = -6

4. Zeroes of p(z) = z^2 -27 are ______ and _______

Ans : $\because$ For zeroes z^2 - 27 = 0

$\Rightarrow$ z^2 -27 $\Rightarrow z = \pm\sqrt{27}$

$\Rightarrow$ $z = \pm 3\sqrt{3}$

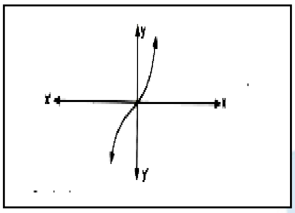
5. Verify that x =3 is a zero of the Polynomial p(x) = $2x^2$- $5x^2$ - 4 x + 3

Ans: Here p(x) = $2x^2$- $5x^2$ - 4 x + 3

p(3) = $2(3)^3$- 5 x –$(-3)^2$ – 4 x 3 + 3

= 54-45 -12 + 3 = 0

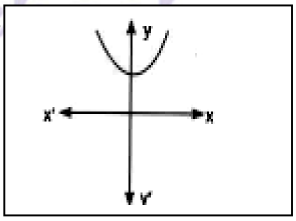
6. The graph of y = f(x) is given below. How many zeroes are there of f(x) ?



Ans : Graph of y= f(x) intersect x -axis in one point only.

Therefore number of zeroes of f(x) is one.

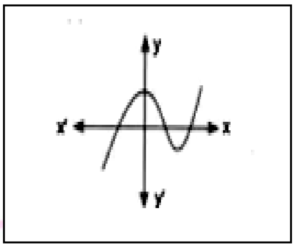
7. The graph of y = f(x) is given how many zeroes are there of f(x) ?



Ans: $\because$ Graph y = f(x) does not intersect x -axis

$\therefore$ f(x) has no zeroes

8. The graph of y= f(x) is given below, for some polynomial f(x).

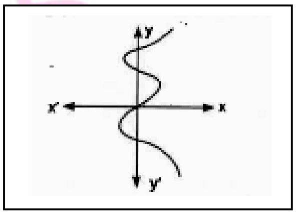


Ans: Find the number of zeroes of f(x)

∵ Graph f(x) intersects x -axis at three different points.

∴ Number of zeroes f(x) =3

9. The graph of x = p(y) is given below, for some polynomial p(y) .



Ans: Find the number of zeroes of p(y)

∵ Graph p(y) intersects y - axis at four different points.

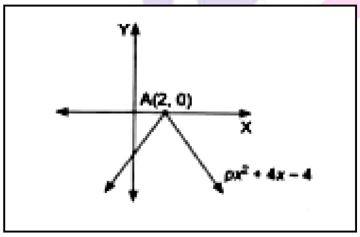
∴ Number of zeroes = 4

10. If one zero of p(x) = ax^2 +b x + c is zero, find the value of c.

Ans: x = 0 is a zero of p(x)

∴ p(0) =0 ⇒ a x $(0)^2$ + b(0) + c =0 ⟹ c =0

11.



Graph of the polynomial p(x) = px^2 + 4 x -4 is given as above. Find the value of p.

Ans: Graph of p(x) touches the x -axis at (2,0)

∴ x =2 is a zero of the p(x)

⟹ p(2) = 0

⟹ $p(2)^2$ +4x2 -4= 0

4p+4 =0

⟹ p=-1

12. If p(x) = ax^2 + bx + c then $-\frac{b}{a}$ is equal to

a) 0 b) 1

c) product of zeroes **d) sum of zeroes**

13. If p(x) = ax^2 +b x +c and a+b+c =0 then one zero is

a) $\frac{-b}{a}$ **b)** $\frac{c}{a}$ c) $\frac{b}{c}$ d) none of these

Ans: p(1) = 0; $a(1)^2$ + b(1) + c = 0 $\Longrightarrow$ and a + b + c =0

$\therefore$ one zero ($\propto$) = 1

14. If p(x) = ax^2+b x +c and a + c = b then one of the zeroes is

a) $\frac{b}{a}$ b) $\frac{c}{a}$ c) $\frac{-c}{a}$ d) $\frac{-b}{a}$

Ans: c) p (-1)= 0 $a(-1)^2$+b(-1)+c = 0

$\Longrightarrow$ a- b + c = 0 $\therefore$ one zero ($\propto$) = -1

$\propto \beta$= product of zeroes = $\frac{c}{a}$ $\Longrightarrow$ (-1) β = $\frac{c}{a}$

$\Longrightarrow$ $\beta = \frac{-c}{a}$

15. The number of polynomials having zeroes as -2 and 5 are

a) 1 b) 2 c) 3 **d) more than 3**

d) $\because$ x^2-3 x -10 2 x^2-6 x -20

$\frac{1}{2}x^2$ - $\frac{3}{2}x$ -5 3 x^2-9 x -30 etc

have zeroes -2 and 5

16. The quadratic polynomials the sum of whose zeroes is- 5 and their product is 6 is

a) x^2 +5 x +6 b) x^2 -5 x +6 c) x^2 -5 x -6 d) $-x^2$ +5 x +6

Ans : sum of zeroes = -5 product 6

Polynomial is

x^2 – (sum of zeroes) x + product of zeroes

$\Longrightarrow$ x^2(-5) x +6 = x^2 +5 x + 6

17. Given that one of the zeroes of the cubic polynomial ax^3 +bx^2 + c x + d is zero the product of the other two zeroes is

a) - $\frac{c}{a}$ **b)** $\frac{c}{a}$ c) 0 d) $\frac{b}{a}$

Ans: $\because$ $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

Let α = 0

So, 0 + $\beta\gamma$ + 0 = $\frac{c}{a}$ $\Rightarrow$ = $\frac{c}{a}$

18. If the product of the zeroes of $x^2 - 3k\,x + 2k^2$ -1 is 7, then value of k are _____ and _____.

Ans: Product of zeroes = 7

$\Rightarrow$ $2k^2$ - 8 = k^2 -4 $\Rightarrow$ k = $\pm$ 2

19. Find the product of the zeroes of-2 $x^2 - k\,x$ +6

Ans: Here a = -2 b=k c = 6

Product of zeroes = $\frac{c}{a}$

i.e $\alpha \times \beta = \frac{6}{-2}$ = -3

20. Find the sum of the zeroes of the given quadratic polynomial -3 $x^2 - k\,x$ +6

Ans: Here a =-3 b = 0, c = k

and sum of zeroes = $\frac{-b}{a}$

i.e. $\alpha + \beta = \frac{-b}{a}$ $\Rightarrow$ $\alpha + \beta = \frac{0}{-3}$ = 0

21. If one zero of the polynomial $x^2 - 4\,x$ + 1 is 2 + $\sqrt{3}$ write the other zero

Ans: Let other zero be α,

$\therefore$ 2 + $\sqrt{3}$ + α = $\frac{b}{a}$ = - $\left(\frac{-4}{1}\right)$

$\Rightarrow$ α =4-2 - $\sqrt{3}$ - 2 -$\sqrt{3}$ = 2 -$\sqrt{3}$

22. Find the zeroes of the polynomial $(x-2)^2$ + 4

Ans: For zeroes $(x-2)^2$ + 4 = 0

$(x-2)^2$ + 2^2=0

Sum of two perfect squares is zero if each of them is zero

$\therefore$ No zero

23. The graph of a quadratic polynomial x^2-3 x -4 is a parabola. Determine the opening of parabola.

Ans: ∵ In x^2- 3 x -4, the Coefficient of x^2 is 1 and 1>0

∴ The parabola opens upwards.

24. If p (x) = x^2+5 x +2 then find p(3)+ p(2)+p(O)

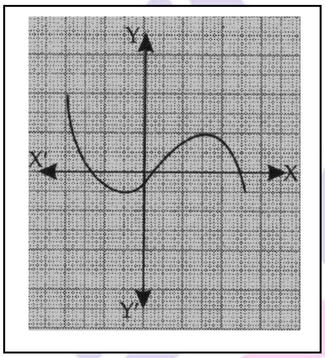
Ans: P(3) = 3^2 + 5(3) +2 = 26

P(2)= 2^2 + 5(2) +2 = 16

P(0)= 0^2 + 5(0) +2 = 2

⇒ P(3) + p(2) + p(0) = 26+16+2+44

25. The graph of y = p(x) is shown in the figure below. How many zeroes does p(x) have.



Ans: Since, the curve (graph) of p(x) is intersecting the x -axis at three points

∵ y = p(x) has 3 zeroes.

26. The coefficient of x and the constant term in a linear polynomial are 5 and -3 respectively. find its zero.

Ans: ∵ The zero of the a linear polynomial

$$= \frac{\text{Constant term}}{\text{Coefficient of } x}$$

∴ The zero of the given linear polynomial

$$= -\frac{(-3)}{5} = \frac{3}{5}$$

27. What is the value of p(x) =x^2 – 3 x -4 at x = -1?

Ans: We have p(x) =x^2 – 3 x -4

∴ P(-1)= $(-1)^2$ – (3(-1))- 4=1+3-4 =0

28. If the polynomial p(x) is divisible by (x -4) and 2 is a zero of p(x), then write the corresponding polynomial.

Ans: Here, p(x) is divisible by (x -4) and also 2 is a Zero of p(x), therefore p(x) is divisible by (x -4) and (x -2)

Thus, the required polynomial p(x) = (x - 4) and (x -2) = x^2-6 x +8

29. What is the zero of 2 x +3?

Ans: ∵ The zero of a linear polynomial

$$= \frac{\text{Constant term}}{\text{Coefficient of } x}$$

∵ The zero of 2x+3 = $\frac{3}{2}$

30. Find the value of p for which the polynomial x^3 + 4x^2 −p x + 8 is exactly divisible by (x -2)

Ans : Here p(x) =x^3 + 4x^2 −p x + 8

∵ (x -2) divides p(x), exactly

⇒ p(2) =0

⇒ $(2)^3$ + 4$(2)^2$ −p(2) + 8 =0

2p =32 ⇒ p=16

31. If α ß are zeroes of the polynomial 2x^2 −5 x + 7, then find the value of α^{-1} + $ß^{-1}$

Ans: Here p(x) = 2x^2 −5 x + 7

α ß are zeroes of p(x)

⇒ $\alpha + ß = \frac{-(-5)}{2} = \frac{5}{2}$ and α ß = $\frac{7}{2}$

∴ $\alpha^{-1} + ß^{-1} = \frac{1}{\alpha} + \frac{1}{ß} = \frac{ß+\alpha}{2\alpha ß} = \frac{\frac{5}{2}}{\frac{7}{2}} = \frac{5}{7}$

32. If p and q are the roots of ax^2 −b x + c = 0, a ≠ 0 then find the value of p+q.

Ans: Here p and q are the roots of ax^2 −b x + c = 0

Sum of roots $= \frac{-b}{a}$

∵ $p + q = \frac{-b}{a}$

33. If -1 is a zero of quadratic polynomial p(x) = kx^2 −5 x - 4 then find the value of k

Ans: Here p(x) = kx^2 −5 x - 4

Since -1 is a zero of p(x)

∴ p (-1) = 0

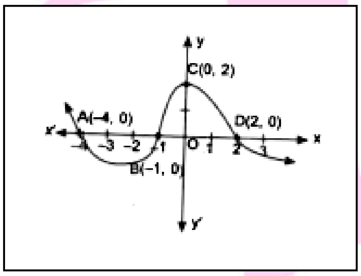
⇒ $k(-1)^2 - 5(-1) - 4 = 0$

⇒ k+5 - 4 = 0

⇒ k =-1

I Short Answer Type Questions

1. Graph of y =f(x) is given below. Find the zeroes of f(x)



Here graph of y=f(x) intersect the x- axis in A (-4, 0) B(-1, 0) and D (2,0)

∴ Zeroes of f(x) are x -coordinates of these points

∴ Zeroes of f(x) are -4,-1 and 2

2. For what value of k, is 3 a zero of the polynomial $2x^2$ + x + k

Since 3 is a zero of the polynomial $2x^2 + x$ + k

∴ p(3) = 0 ⟹p(x) = $2x^2 + x$ + k

$\Rightarrow$ p(3) = $2(3)^2$ + 3 + k

$\Rightarrow$ 0 = 18 + 3 +k $\Rightarrow$ k = -21

3. Find the zeroes of $\sqrt{3}x^2$ + $10x$ +$7\sqrt{3}$

$\sqrt{3}x^2 + 10x + 7\sqrt{3}$

$= \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3}$

$= \sqrt{3}x\,(x + \sqrt{3}) + 7\,(x + \sqrt{3})$

$= (\sqrt{3}x + 7)\,(x + \sqrt{3})$

For zeroes of the polynomial

$= (\sqrt{3}x + 7)\,(x + \sqrt{3}) = 0$

$\Rightarrow$ $\sqrt{3}x + 7 = 0$ or $x + \sqrt{3} = 0$

$\Rightarrow$ $\sqrt{3}x = -7$ or $x = -\sqrt{3}$

$\Rightarrow$ $x = \frac{7}{\sqrt{3}}, -\sqrt{3}$

4. Find a quadratic polynomial whose zeroes are -9 and $-\frac{1}{9}$

Sum of zeroes = -9 and $\left(-\frac{1}{9}\right) = \frac{-81-1}{9} = \left(\frac{-82}{9}\right)$

Product of zeroes= $(-9) \times \left(-\frac{1}{9}\right) = 1$

Quadratic polynomial = x^2 - (sum of zeroes) x + product of zeroes

$= x^2 - \left(\frac{-82}{9}\right) x + 1 = 9x^2 + 82x + 9$

5. If the sum of zeroes of the quadratic polynomial ky^2+ 2y-3k is equal to twice their product find the value of k.

P (y) =ky^2+ 2y-3k

a=k, b=2 c=-3K A.T.O Sum of zeroes = 2 x product of zeroes

$\Rightarrow \quad \frac{-b}{a} = 2 \times \frac{c}{a} \Rightarrow \frac{-2}{k} = 2 \times \frac{-3k}{k}$

$\Rightarrow \quad \frac{2}{k} = 6 \Rightarrow \quad k = \frac{1}{3}$

6. If zeroes of p(x) = ax^2+ bx +c are negative reciprocal of each other, find the relationship between a and c

p(x) = ax^2+bx+c

Let one zero = $\propto$

$\therefore$ Other zero = $-\frac{1}{\propto}$

Now product of zeroes = $\frac{c}{a}$

$\Rightarrow \quad \propto \times -\frac{1}{\propto} = \frac{c}{a} \Rightarrow \frac{c}{a} = -1$

$\Rightarrow$ c = -a or a + c =0

7. Find the quadratic polynomial whose sum of zeroes is 8 and their product is 12. Hence find zeroes of polynomial.

Let α, β be zeroes of polynomial

Now here $\alpha + \beta$ = 8 $\alpha\beta$ = 12

Required polynomial

p(x) = k[$x^2 - (\alpha + \beta)\ x + \alpha\beta$]

k is a constant

$\Rightarrow$ p(x) = k[$x^2 - 8\ x + 12$]

In particular taking k = 1

Reqd. polynomial = $x^2 - 8\ x + 12$

Now p(x) = $x^2 - 6\ x - 2\ x + 12$

= x(x - 6) -2 (x - 6)

= (x-6) (x-2)

$\therefore$ p($x - 6$) = 0 and p(x-2) = 0

$\Rightarrow$ x = 6, 2 thus zeroes of polynomial are 6 and 12

8. Check whether $x^2+3\ x$ +1 is a factor of $3x^4+5x^3$ -7x^2 +2 x+2

Let p(x) = $3x^4+5x^3$ -7x^2 +2 x+2, g(x) = x^2 +3 x+1

Next we divide p(x) by g(x)

x^2 +3 x+1) $3x^4+5x^3$ -7x^2 +2 x+2 ($3x^2$ -4 x+2

$3x^4+5x^3$ -7x^2

\- \- \-

$-4x^3$ -10x^2 +2 x+2

$-4x^3$ -12x^2 -4 x

\+ \+ \+

2x^2 + 6 x+ 2

2x^2 + 6 x+ 2

\- \- \-

0

Using division algorithm

$3x^4+5x^3$ -7x^2 +2 x+2 (x^2 +3 x+1) (3 x^2 - 4 x+2) + 0 = (x^2 +3 x+1)

(3 x^2 - 4 x+2)

Clearly as remainder is 0 so the divisor x^2 +3 x+1 appear on R.H.S. as factor of

$3x^4+5x^3$ -7x^2 +2 x+2

9. If one zero of the polynomial (a^2 + 9) x^2+ 13x +6a is reciprocal of the other, find the value of a .

Let $\alpha, \frac{1}{\alpha}$ be the zeroes of (a^2 + 9) x^2+ 13x +6a

Product of zeroes = $\frac{6a}{a^2+9}$

$\Rightarrow$ $\alpha \times \frac{1}{\alpha} = \frac{6a}{a^2+9}$ $\Rightarrow$ a^2 +9 -6a =0

$(a-3)^2$ = 0

$\Rightarrow$ a = 3

10. If α and β are zeroes of x^2+ 7x +12 then find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$ -2 $\alpha\ \beta$

Here $\alpha+\beta$ = -7 $\alpha\ \beta$ = 12

$\frac{1}{\alpha}+\frac{1}{\beta}$ -2 $\alpha\ \beta$ = $\left|\frac{\beta+\alpha}{\alpha\beta}\right|$ - 2 $\alpha\ \beta$

= $\frac{-7}{12}$ -2(12) = $\frac{-7}{12}$ -24

= $\frac{-7-288}{12}$ = $\frac{-295}{12}$

11. Find $\alpha^{-1}+\beta^{-1}$ if α and β are zeroes of the polynomial $9x^2$-3x – 2

Ans: Since α and β are zeroes of p(x) = $9x^2$-3x – 2

$\therefore \quad \alpha+\beta = \frac{-(-3)}{9} = \frac{1}{3}$, $\alpha\beta = \frac{-2}{9}$

$\alpha^{-1}+\beta^{-1} = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{\frac{1}{3}}{\frac{-2}{9}} = \frac{-3}{2}$

12. Find whether $2x^3$ -1 is a factor of $2x^5+10x^4$ +$2x^2$ +5x+1 or not

$2x^3$ -1) $2x^5+10x^4+0x^3$ + $2x^2$ +5x+1 (x^2 + 5x

$\pm 2x^5 \qquad \mp x^2$

$10x^4$ +$3x^2$ +5x+1

$\pm 10x^4 \qquad \mp 5x$

$3x^2$ +10x+1 ← r(x)

Since r(x)≠ 0

$\therefore$ $2x^3$ -1 is not a factor of given polynomial

13. If α, β, γ are zeroes of the polynomial f(x) = x^3 -3x^2 +7x-12 then find the value of $((\alpha\beta)^{-1}+(\beta\gamma)^{-1}+(\gamma\alpha)^{-1})$

Here $\alpha+\beta+\gamma$ = $\frac{-(-3)}{1}$ = 3

and $\alpha\ \beta\ \gamma$ = $\frac{-(-12)}{1}$ = 12

Now $((\alpha\beta)^{-1}+(\beta\gamma)^{-1}+(\gamma\alpha)^{-1}) = \frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$

$= \frac{\gamma+\alpha+\beta}{\alpha\beta\gamma} = \frac{3}{12} = \frac{1}{4}$

14. For what value of k is the polynomial x^3 - kx^2 +3x-18 is exactly divisible by (x-3)

If p(x) = x^3 +kx^2 +3x-18 is exactly divisible by (x-3)

⇒ p(3) = 0 ⇒ $(3)^3$ + k$(3)^2$ +3 (3)-18 = 0

⇒ 9k = -18 ⇒ k = -2

II Short Answer Type Questions

1. Find the value of k such that the polynomial x^2+(k +6) x +2 (2k-1) has sum of its zeroes equal to half of their product.

The given polynomial is x^2+ (k +6) x + 2 (2k-1)

Let α and β be the zeroes of polynomial

$$\alpha + \beta = \left|\frac{-(k+6)}{1}\right| = k + 6$$

$$\alpha\beta = \frac{2(2k-1)}{1} = 4k - 2$$

$$\therefore \quad \alpha + \beta = \frac{1}{2}\alpha\beta$$

$\Rightarrow$ k + 6 = $\frac{1}{2}$(4k-2)

$\Rightarrow$ 2k +12 = (4k-2)

$\Rightarrow$ 2k = 14 $\Rightarrow$k=7

2. If one root of the quadratic polynomial 2 x^2-3 x +p is 3, find the other root. Also find the value of p.

$\because$ 3 is a root (zero) of p(x)

$\Rightarrow$ $2(3)^2$ -3x 3 + p = 0

$\Rightarrow$ $18 - 9 + p$=0 $\Rightarrow$ p = -9

Now p(x) =2 x^2-3x-9 = 2x^2-6 x + 3 x -9

=2x(x -3) +3(x -3)

= (x -3) (2 x +3)

For roots of polynomial p(x) = 0

$\Rightarrow$ (x -3) (2 x +3) = 0

$\Rightarrow$ (x -3) or x = $\frac{3}{2}$ other root = $-\frac{3}{2}$

3. If α and β are zeroes of the quadratic polynomial $4x^2+4x+1$ then form a quadratic polynomial whose zeroes are 2α and 2β

$P(x) = 4x^2+4x+1$

$\because$ α, β are zeroes of $p(x)$

$\therefore \alpha + \beta$ = sum of zeroes $= \frac{-b}{a}$

$\Rightarrow$ $\alpha + \beta = \frac{-4}{4} = -1$ _____(i)

Also α, β product of zeroes of $= \frac{c}{a}$

$\Rightarrow$ $\alpha, \beta = \frac{1}{4}$ _____(ii)

Now a quadratic polynomial whose zeroes are 2α and 2β

$= x^2 - (2\alpha + 2\beta)x + 2\alpha \times 2\beta$

$= x^2 - 2(\alpha + \beta)x + 2\alpha + 4(\alpha\beta)$

$= x^2 - 2 \times (-1)x + 4 \times \frac{1}{4}$ [Using eq.(1) and (ii)]

$= x^2 + 2x + 1$

4. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Here $p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

For zeroes of p(y), p(y) = 0

$\Rightarrow$ $7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$

$\Rightarrow$ $21y^2 - 11y - 2 = 0$

$\Rightarrow$ $21y^2 - 14y + 3y - 2 = 0$

$\Rightarrow$ $7y(3y-2) + 1(3y-2) = 0$

and $(7y+1)(3y-2) = 0$

$\Rightarrow$ y = $\frac{-1}{7}$, $\frac{2}{3}$

∴ zeroes are $\frac{-1}{7}$ and $\frac{2}{3}$

Also a = 7 b = $\frac{-11}{3}$, c = $\frac{-2}{3}$

$\Rightarrow$ Sum of zeroes = $\frac{-1}{7}$ + $\frac{2}{3}$ = $\frac{-3+14}{21}$ = $\frac{11}{21}$

Also $\frac{-b}{a}$ = $\frac{-(-11/3)}{7}$ = $\frac{11}{21}$

$\Rightarrow$ Sum of zeroes = $\frac{-b}{a}$

and product of zeroes = $\frac{-1}{7}$ x $\frac{2}{3}$ = $\frac{-2}{21}$

Also $\frac{c}{a}$ = $\frac{\frac{-2}{3}}{7}$ = $\frac{-2}{21}$

$\Rightarrow$ Product of zeroes = $\frac{c}{a}$

5. If the zeroes of $x^2 - px + 6$ are in the ratio 2:3 find p

p(x) =$x^2 - px + 6$

Let zeroes are 2m and 3m

Sum of zeroes = - $\frac{b}{a}$

$\Rightarrow$ 2m + 3m= $\frac{-(-p)}{1}$

$\Rightarrow$ 5m = p .....(i)

Product of zeroes = $\frac{c}{a}$

$\Rightarrow$ 2m x 3m = $\frac{6}{1}$

$\Rightarrow$ $6m^2$ = 6

$\Rightarrow$ m^2=1

$\Rightarrow$ $m = \pm 1$ When m = 1, eq (i) becomes

5x1 = p

$\Rightarrow$ p = 5

When m = -1, eq (i) becomes

5x -1 = p

$\Rightarrow$ p= -5

$\therefore$ p= ± 5

6. If α, β are the zeroes of polynomial p(x) = x^2-k(x +1)-p such that $(\alpha+1)(\beta+1)=0$ find p

p(x) =$x^2 - kx - k - p$=0

a=1, b = -k, c = -k-p

$\because$ α, β are zeroes of p(x)

$\therefore$ $\alpha+\beta$ = $-\frac{b}{a} \Rightarrow$ $\alpha+\beta$ = k

and $\alpha\beta$ = $\frac{c}{a}$ $\Rightarrow$ $\alpha\beta$ = -k -p

Also $(\alpha+1)$ $(\beta+1)$ = 0

$\Rightarrow$ $\alpha\beta$ + α + β+1 = 0

$\Rightarrow$ (-k-p) + k + 1 = 0

-p + 1 = 0

$\Rightarrow$ p = 1

7. a, b, c are co-prime a≠1 such that 2b= a + c. If $ax^2 - 2bx + c$ and $2x^2 - 5x^2 + kx$+4 has one integral root common, then find the value of k.

$p(x) = ax^2 - 2bx + c$

P(1) = $= a(1)^2 - 2b \times 1 + c$

= a-2b+c

= a + c - 2b

Given a +c =2b

∴ p(1) = 2b - 2b = 0

⇒ x = 1 is a zero of p(x)

Now product of zeroes of p(x) = $\frac{c}{a}$

Other root $\frac{\frac{c}{a}}{1}$ = $\frac{c}{a}$

Roots are 1 and $\frac{c}{a}$

∵ $\frac{c}{a}$ are co-prime

∴ integral root of p(x) =1

A.T.O. 1 is a root of $f(x) = 2x^3 - 5x^2 + kx + 4$

⇒ f(1) =0

⇒ $2(1)^3 - 5(1)^2 + k$x1 + 4=0

K = -1

8. Find all the zeroes of $2x^4 - 13x^3 + 19x^2 + 7x - 3$, if you know that two of its zeroes are $2+\sqrt{3}$ and $2-\sqrt{3}$

Given, $x = (2+\sqrt{3})$ and $x = (2-\sqrt{3})$ are zeroes of p(x) = $2x^4 - 13x^3 + 19x^2 + 7x - 3$

∴ $(x - (2+\sqrt{3}))(x - (2-\sqrt{3}))$ is factor of p(x)

⇒ $(x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$ is a factor of p(x)

= $(x-2)^2 - (\sqrt{3})^2$

= $x^2 - 4x + 4 - 3$

= $x^2 - 4x + 1$

Now we divide p(x) by x^2-4x+1

$$\begin{array}{r|l|l} x^2-4x+1 & 2x^4 \ -13x^3 + 19x^2 + 7x - 3 & 2x^2-5x-3 \\ & 2x^4 \ -8x^3 + 2x^2 & \\ & - \quad + \quad - & \\ \hline & -5x^3 + 17x^2 + 7x - 3 & \\ & -5x^3 + 20x^2 - 5x & \\ & + \quad - \quad + & \\ \hline & -3x^2 + 12x - 3 & \\ & -3x^2 + 12x - 3 & \\ & + \quad - \quad + & \\ \hline & x & \\ \hline \end{array}$$

Now p(x) = (x^2-4x+1) (2 x^2-5x-3)

∴ Other zeroes are given by

$2x^2-5x-3 = 0$

⇒ $2x^2-6x + x-3 = 0$

⇒ $2x(x-3) +1 (x-3) = 0$

⇒ $(2x+1)(x-3) = 0$

⇒ $2x+1=0$ or $x-3 = 0$

$x = -\frac{1}{2}, 3$

∴ Zeroes of given polynomial are $-\frac{1}{2}$, 3 (2+ $\sqrt{3}$) (2- $\sqrt{3}$)

9. Find all the zeroes of $2x^4-3x^3 -3x^2 + 6x-2$, if it is given that two of its zeroes are 1and $\frac{1}{2}$

Given x=1 x = $\frac{1}{2}$ are zeros of p(x)= $2x^4 -3x^3 -3x^2+6x-2$

∴ $(x - 1)(x - \frac{1}{2})$ or $(2x - 1)$ are factor of p(x)

⇒ $(x - 1)(2x - 1) = 2x^2-3x+1$

$2x^2$-3x+1) $2x^4$ -3x^3 -3x^2+6x-2 (x^2-2

$2x^4$ -3x^3 + x^2

- + -

-4x^2+6x-2,

-4x^2+6x-2

+ - +

0

∴ p(x) =(2x^2-3x+1) (x^2-2) = 0

Other zeros are given by (x^2-2) = 0

⇒ $x^2 = 2 \Rightarrow x \pm \sqrt{2}$

∴ Zeros of p(x) are =$\sqrt{2}$, $\frac{1}{2}$, 1, $\sqrt{2}$.

10. Find all zeroes of the polynomial 3x^3 +10x^2-9x-4

Since, 1 is a zero of p(x)

Therefore, (x-1) is a factor of p(x)

Dividing p(x) by (x-1), we have

x-1) 3x^3 +10x^2-9x -4 (3x^2+13x +4

$3x^3 - 3x^2$

- +

$13x^2 - 9x - 4$

$13x^2 - 13x$

4x -4

4x -4

- +

0

∴ By Division Algorithm,

p(x) = (3x^2+13x +4) (x-1)

∴ Zeroes of p(x) are given by p(x) = 0

⇒ (3x^2+13x + 4) (x -1) = 0

⇒ (3x (x + 4) +1 (x + 4) (x -1) = 0

$\Rightarrow \quad (x + 4)(3x + 1)(x - 1) = 0$

$\Rightarrow \quad (x + 4) = 0 \Rightarrow x = -4$

Or $3x + 1 = 0$ Or $x - 1 = 0 \Rightarrow x = 1$

$x = -\frac{1}{3}$

$\therefore$ Zeroes of $p(x)$ are $x = -4, -\frac{1}{3}, 1$

11. Divide the polynomial $3x^3 - 6x^2 - 20x + 14$ by the polynomial $x^2 - 5x + 6$ and verify the division algorithm.

$$\begin{array}{r|l|l} x^2 - 5x + 6 & 3x^3 - 6x^2 - 20x + 14 & 3x + 9 \\ & 3x^3 - 15x^2 + 18x & \\ & - \quad + \quad - & \\ \hline & 9x^2 - 38x + 14 & \\ & 9x^2 - 45x + 14 & \\ & - \quad + \quad - & \\ \hline & 7x - 40 & \end{array}$$

By Division Algorithm

$3x^3 - 6x^2 - 20x + 14 = (x^2 - 5x + 6)(3x + 9) + (7x - 40)$ or

$p(x) = q(x)\, g(x) + r(x)$

12. If α and β are the zeroes of a quadratic polynomial $(x^2 - x - 2)$ then find the value of $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)$

Comparing $(x^2 - x - 2)$ with $ax^2 + bx + c$ we have a=1, b = -1 c=-2

$\therefore \quad \alpha + \beta = \frac{-b}{a} = \frac{-(-1)}{1} = 1$

and $\quad \alpha\beta = \frac{c}{a} = \frac{(-2)}{1} = -2$

Now $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta - \alpha}{\alpha\beta} = \frac{-[\alpha - \beta]}{\alpha\beta}$

[$\therefore \quad (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$(1)^2 - 4(-2) = 1 + 8 = 9$

$\therefore (1)^2 - 4(-2) = 1 + 8 = 9$

$\therefore \quad \alpha - \beta = \sqrt{9}$

$\Rightarrow \quad \alpha - \beta = \pm 3]$

$= \frac{-(\pm 3)}{-2} = \frac{-3}{2}$ and $\frac{3}{2}$

Thus, $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{3}{2}$ or $\frac{-3}{2}$

13. On dividing p(x) by a polynomial x - 1- x^2, the quotient and remainder were (x -2) and 3 respectively. Find p(x)

Here

dividend = p(x)

Divisor, g(x) = (x - 1- x^2)

Quotient q(x) = (x -2)

Remainder r(x) = 3

$\therefore$ Dividend = [Divisor x Quotient] + Remainder

$\therefore$ p(x) = [g(x) x q(x) + r(x)]

= [(x- 1- x^2) (x-2) + 3

=[(x^2- x - x^3-2 x + 2 +2x^2]+3

= 3 x^2- 3 x - x^3 + 2+3

= -x^3 + 3 x^2-3 x +5

14. Find the zeroes of the quadratic polynomial 5 x^2-4-8 x and verify the relationship between the zeroes and the coefficients of the polynomial.

p(x) = 5 x^2- 4 - 8 x =5 x^2- 8 x -4

= 5 x^2- 10x + 2x -4

= 5 x (x -2) +2 (x- 2)

= (x-2) (5 x +2)

= 5 (x -2)$\left(x + \frac{2}{5}\right)$

$\therefore$ Zeroes of p(x) are 2 and -$\frac{2}{5}$

Relationship Verification

Sum of the zeroes = - $\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$

⇒ $2 + \left(-\frac{2}{5}\right) = \frac{-(-8)}{5}$

⇒ $\frac{10-2}{5} = \frac{8}{5}$

⇒ $\frac{8}{5} = \frac{8}{5}$

i.e. LHS = RHS

⇒ relationship is verified

Product of the zeroes = $\frac{Constant\ term}{Coefficient\ of\ x^2}$

⇒ $2 \times \left(-\frac{2}{5}\right) = \frac{(-4)}{5}$

⇒ $\frac{-4}{5} = \frac{-4}{5}$

i.e. LHS = RHS

⇒ The relationship is verified.

I. Long answer choice questions

1. If the polynomial $x^4+2x^3+8x^2+12x+18$ is divided by another polynomial x^2+5 the remainder comes out to be p x + q. find the values of p and q

$$
\begin{array}{r|l}
 & x^2+2x+3 \\
\hline
x^2+5 & x^4+2x^3+8x^2+12x+18 \\
 & x^4 \quad\quad +5x^2 \\
 & - \quad\quad - \\
\hline
 & 2x^3+3x^2+12x+18 \\
 & 2x^3+ \quad +10x \\
 & - \quad\quad - \\
\hline
 & 3x^2+2x+18 \\
 & 3x^2 \quad\quad +15 \\
 & - \quad\quad - \\
\hline
 & 2x+3
\end{array}
$$

∴ Remainder = $2x+3$

Comparing $2x+3$ with p x + q we have

p=2 and q=3

2. If α, β, γ are the zeroes of x^3-7x^2+11x -7 find the value of

(i) $\alpha^2 + \beta^2 + \gamma^2$ (ii) $\alpha^3 + \beta^3 + \gamma^3$

Comparing p(x) = x^3+bx^2+11x -7 with standard cubic polynomial ax^3-bx^2+cx +d.

we have a=1 b-7, c =11, d =-7

$\therefore$ $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-7)}{1} = 7$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{11}{1} = 11$

$\alpha, \beta, \gamma = \frac{-d}{a} = \frac{-(-7)}{1} = 7$

i) Now since $(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$\Rightarrow$ $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 = 2(\alpha\beta + \beta\gamma + \gamma\alpha) = (7)^2 - 2(11) = 49 - 22 = 27$

ii) Since

$\alpha^2 + \beta^2 + \gamma^2 - 3\alpha\beta\gamma = (\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)(\alpha + \beta + \gamma)$

$= [(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha) - (\alpha\beta + \beta\gamma + \gamma\alpha)](\alpha + \beta + \gamma)$

$\Rightarrow$ $\alpha^3 + \beta^3 + \gamma^3 = [(\alpha + \beta + \gamma)^2 - 3(\alpha\beta + \beta\gamma + \gamma\alpha)(\alpha + \beta + \gamma) + 3\alpha\beta\gamma$

= $[7^2 - 3(11)]$ 7+ 3 x7

= (49-33) 7 + 21

= 16 x 7 + 21= 133

3. Find the quadratic polynomial whose zeroes are 1 and -3. Verify the relation between the coefficients and the zeroes of the polynomial

$\because$ The given zeroes are 1 and -3

$\therefore$ Sum of the zeroes =1+ (-3) = -2

Product of the zeroes 1x (-3) = -3

A quadratic polynomial p(x) is given by

x^2- (sum of the zeroes) x + (product of the zeroes)

$\therefore$ The required polynomial is

x^2-(-2)x +(-3)

$\Rightarrow$ x^2+2x-3

Verification of relationship

∵ Sum of the zeroes = $\frac{[Coefficient\ of\ x]}{Coefficient\ of\ x^2}$

∴ $1 + (-3) = \frac{-[2]}{1}$

⇒ $-2 = -2$

i.e. LHS = RHS

⇒ The sum of the zeroes is verified

∵ Product of the zeroes = $\frac{[Constant\ term]}{Coefficient\ of\ x^2}$

∴ $1 \times (-3) = \frac{[-3]}{1}$

⇒ $-3 = -3$

i.e. L.H.S. = R.H.S.

⇒ The product of zeroes is verified

4. What must be added to $6x^5 + 5x^4 + 11x^3 - 3x^2 + x + 1$, so that the polynomial so obtained is exactly divisible by $3x^2 - 2x + 4$?

$3x^2 - 2x + 4 \;)\; 6x^5 + 5x^4 + 11x^3 - 3x^2 + x + 1$

$6x^5 - 4x^4 + 8x^3$

$- \quad + \quad -$

$9x^4 + 3x^3 - 3x^2 + x + 1$

$9x^4 - 6x^3 + 12x^2$

$+ \quad + \quad -$

$9x^3 - 15x^2 + x + 1$

$9x^3 - 6x^2 + 12x$

$- \quad + \quad -$

$-9x^2 - 11x + 1$

$-9x^2 + 6x - 12$

$+ \quad - \quad +$

$-17x + 13$

Therefore, we must add $-(-17x + 13)$

i.e. $17x - 13$

5. Find the value of b for which the polynomial $2x^3+9x^2-x-b$ is divisible by $2x+3$

$$
\begin{array}{r|l}
 & x^2+3x-5 \\
2x+3 & 2x^3+9x^2-x-b \\
 & 2x^3+3x^2 \\
 & -\quad - \\
\hline
 & 6x^2-x-b \\
 & 6x^2+9x \\
 & -\quad - \\
\hline
 & -10x-b \\
 & -10x-15 \\
 & +\quad + \\
\hline
 & 15-b
\end{array}
$$

Polynomial $2x^3+9x^2-x-b$ divisible by $2x+3$ then the remainder must be zero

So, 15-b =0

$\Rightarrow$ b=15

6. Find the zeroes of a cubic polynomial p(x)= $3x^3$-$5x^2-11x-3$ when it is given that product of two of its zeroes is -1

Here, p(x) = $3x^3$-$5x^2-11x-3$ On comparing p(x) with ax^3+bx^2+cx+d, we have

a =3, b =- 5, c =-11, d = -3

Let α, β, γ be the zeroes of the given polynomial

$\therefore$ $\alpha+\beta+\gamma = \frac{-b}{a} = \frac{-(-5)}{3} = \frac{5}{3}$.......(i)

$\Rightarrow$ $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a} = \frac{-11}{1}$ ....(ii)

$\alpha, \beta, \gamma = \frac{-d}{a} = \frac{-(-3)}{3} = 1$ ....(iii)

Let product of α and β be given as -1

i.e. $\alpha\beta = -1$ ....(iv)

(iii) $\Rightarrow$ (-1) $\gamma = 1$ $\Rightarrow \gamma = -1$ .... (v)

From (i) $\alpha+\beta+(-1) = \frac{5}{3}$

$\alpha+\beta = \frac{5}{3} + 1 = \frac{8}{3}$ ....(vi)

From (iv) $\beta = \frac{-1}{\alpha}$

Putting $\beta = \frac{-1}{\alpha}$ in (vi) we get

$$\alpha - \frac{1}{\alpha} = \frac{8}{3}$$

Or $\frac{\alpha^2-1}{\alpha} = \frac{8}{3}$

$\Rightarrow \quad 3\alpha^2 - 3 = 8\alpha$

$\Rightarrow \quad 3\alpha^2 - 8\alpha - 3 = 0$

$\Rightarrow \quad 3\alpha^2 - 9\alpha + \alpha - 3 = 0$

$3\alpha\ (\alpha-3)(3\alpha+1) = 0$

$\Rightarrow \quad \alpha = 3, \frac{-1}{3}$

When $\alpha = 3\ \beta = \frac{-1}{\alpha} = \frac{-1}{3}$

and when $\alpha = 3\ \ \beta = \frac{-1}{\alpha} = \frac{-1}{\frac{-1}{3}} = 3$

} Same pair is obtained in each case

Hence zeroes of the polynomial are -1, 3 , $\frac{-1}{3}$